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Abstract

This paper develops a market microstructure model with asymmetric information in order to quantify the influence which practical decision rules have on asset prices. The users of practical decision rules have incomplete information at their disposal and trade in a market with both fully informed and uninformed investors, as well as with a competitive market maker. The users of practical decision rules affect the periodical ask and bid prices in two ways: by means of the precision of their information and through their share in the totality of investors, respectively. The resulting bid-ask spread is positive and proportional to the c.p. variation of these two influencing factors and is attributable to adverse selection costs.

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1 Introduction

For the last thirty years the academic community has been engaged in a considerable debate about the efficiency of the financial markets, but has not yet reached a satisfactory conclusion. In the meantime, the traders' community has kept investing according to some well-known practical decision rules (such as fundamental indicators, e.g. book-to-market values, price-earnings or price-cash-flows ratios, and technical patterns, respectively, derived from the systematical observation of the prices' and trading volume's evolution). Despite the growing interest with regard to the application of such empirical methods, their efficiency as a support in decision-making as well as their consequences still remain controversial and unsatisfactory when investigated from a theoretical viewpoint.

Two different aspects help reveal the implications of the intensive use of practical decision rules for asset prices: a qualitative one, derived from their informational accuracy, and a quantitative one, generated through the proportion of their users to the entire pool of investors.

1.1 Overview

The **purpose** of this paper consists in quantifying the influence of these two factors on price formation. Therefore, we model the transaction prices as functions of variables standing for the quality and the quantity of the information derived by means of practical decision rules. Our simple framework accounts for informational asymmetries among the market participants, acting as a driving force of the particular price-setting process.

There are numerous related theoretical and empirical studies, investigating the implications of the informational asymmetry on transaction prices. Some of the most **recent approaches** such as Garmaise and Moskowitz (1999), Van Ness, Van Ness and Warr (2001) or Hanousek and Podpiera (2003) attempt to measure the spread component due to the informational asymmetry on account of real market data. Other empirical results, e.g. those delivered by Easley, Hvidkjaer and O'Hara (2002) stress the significance of information-based trading on the price evolution and on the expected returns. Garleanu and Pedersen (2002) demonstrate theoretically the indirect effect of the spread component due to adverse selection on the expected returns (i.e. this component generates distortions of the trade decisions).[1] In conjunction with our approach accounting for the two-fold influence of the supporters of the practical decision rules, Easley and O'Hara (2001) show that both the quantity and the quality of the information affect asset prices.[2]

The present **model focuses** on the process of price formation, regarded as a result of the information, actions and assessments of certain different categories of market participants, namely:

- the perfectly informed investors - a small group of fully rational traders delivering exact forecasts;
- the imperfectly informed investors - a separate category of not fully rational users of practical decision rules;
- the noise traders - a group of irrational uninformed traders;

[1] For a summary of the previous empirical and theoretical research regarding the informational asymmetry and the bid-ask spread, see Coughenour and Shastri (1999), Madhavan (2000) and Biais, Glosten and Spatt (2002).

[2] Due to the fact that private information increases the systematic risk of the asset holding, the investors expect higher returns for the stocks characterized by more private information. The companies can thus reduce their capital costs by influencing the quality and the quantity of the information released.

- furthermore, one market maker undertakes the periodical price setting in a competitively manner, and the subsequent execution of the orders of these three groups of investors, respectively.

At any moment of a given trade period investors can either buy or sell only one unit of a risky asset. The value of this asset indirectly signalizes the current economic situation and is modelled as a binary random variable (with two values: high and low). Due to the inability to directly observe the value of the risky asset, traders try to estimate it by means of the available information. Accordingly, the periodical decision to release buy or sell orders draws upon this information, which differs from one investor category to the other. The market maker collects the forwarded orders and fixes the transaction prices (one for the buy; the ask, the other for the sell; the bid), according to her own assessment regarding the value of the risky asset.

This paper's **contribution** to the better understanding of the influence of practical decision rules on asset prices consists of several aspects. First, the users of such empirical methods are regarded as imperfectly informed and modelled as a separate category of investors. Second, we emphasize and quantify their twofold price impact: qualitatively (i.e. as the informational precision of the practical decision rules, expressed as the probability of inferring informative signals which indeed correspond to the real economic situation), and quantitatively (i.e. as the fraction of supporters of such empirical methods in respect of the entire number of investors). The prices are therefore fixed on the basis of the market maker's estimation of the probability that the investors want to buy or sell. This assessment incorporates both the probability of deciding to buy or to sell (ascribed to the informational precision), and the weight of each category of investors.

Several interesting **results** emerge from the analysis. On the one hand, the adverse selection the market maker is exposed to entails the appearance of a positive bid-ask spread, independent of the values of the model variables. This result was previously demonstrated in other approaches and thus serves as a confirmation of the model's validity.

On the other hand, we concentrate on the influence of the users of practical decision rules on prices and therefore on the bid-ask spread. In this context, we demonstrate that the more exact the information generated by empirical decision rules is (other things being equal), the more unfavorable the transaction terms for all traders in the market are (namely the higher the buy and the lower the sell price). The bid-ask spread rises accordingly with an increasing speed. A higher fraction of imperfectly informed traders (other things being equal) similarly sway the market maker to worsen the prices. For low proportions of the users of empirical trading rules the bid-ask spread variation is not very sensitive to the modification of this proportion, but it becomes extreme for high values of the fraction of these market participants. A possible explanation for these c.p. effects relies on the adverse selection costs generated by the informational asymmetry between the traders and the market maker. The latter protects herself against the losses from trading with better informed counter-parties by increasing the bid-ask spread.

Finally, we undertake a comparison of the perceptions of the perfectly and imperfectly informed traders with regard to the transaction prices. Deviations of price perception can determine real price distortions, as demonstrated by De Long, Shleifer, Summers and Waldmann (1990) and Easley and O'Hara (2001).[3] We call the perception bias occurring for the users of practical decision rules "misperception". The misperception can be expressed in two ways: by means of the assessments of the current economic situation (when the users of practical decision rules derive the same signal as the benchmark, namely the perfectly informed traders), and with the aid of the decision made in a given economic situation,

[3] Section 4.3 emphasizes the differences between the assumptions of De Long, Shleifer, Summers and Waldmann (1990) and this approach.

respectively. As expected, both specifications of misperception decrease with an increasing quality of the information derived from the practical rules, but their variation takes different formes. A close analysis of the potentially direct connection between the misperceptions and the prices could constitute an interesting topic for further research.

1.2 Delimitation of the model

The concept of this approach relies on several related classical market microstructure models, such as Glosten and Milgrom(1985), Kyle (1985) or Easley and O'Hara (1987), which explain the bid-ask spread by means of the informational asymmetry dominating the market. There are nevertheless some differences between these approaches and our model, as well as some other points which deserve to be mentioned.

In the present model, market orders are simultaneously (and not sequentially) released. Consequently, investors do not learn by observing the contemporaneous actions of other agents and thus by trying to infer the information underlying them. The periodical learning process can be ascribed only to collective experience (consisting of past data), and (only for informed investors) to new information. The prices reflect the new information on the basis of the assessment of the market maker concerning the economic situation.

Furthermore, the risky asset does not exhibit a constant "true" value, but merely a variable one, by representing an indicator of the economic situation, which can fluctuate from one trading moment to the other. Correspondingly, the perfectly informed investors do indeed receive new information in every period. The other (myopic) agents do not consequently have the time and opportunity to deduce the state of the world from the actions of the fully informed traders.

In addition, trading is structured in such a manner (cf. Kyle (1985)), that prices are fixed only after the orders have been placed, so that these prices show no relevance for the actual investors' decisions.

Because a free choice of the order size can result in multiple equilibria (either when this order size varies discreetly, as in Easley and O'Hara (1987) or continuously, as in Kyle (1985)), our model further allows only for a restricted trade size (of one unit per person and period) (as in Glosten and Milgrom (1987).

Moreover, quite contrary to Kyle (1985), there is no group of agents exercising strategic influence on the prices in the current approach. Accordingly, we cannot check for periodical interconnections between the agents' decisions and the prices.

The remainder of this **paper** is **organized** as follows. In Section 2 we set out the basic notations, definitions and assumptions. Section 3 gives account of the most important trading steps, beginning with a general description of the trading course and continuing for each trader category with the specification of the trading rules, the formation of assessments with regard to the economic situation and finally the calculation of expected earnings. Section 4 discusses the main results of the model, these being the periodical price calculation and the influence of users of practical trading rules on the prices. The final Section summarizes the most important conclusions and describes some possible extensions. The proof of the main results and several graphics illustrating these are contained in the Appendix.

2 Variables and Assumptions

1. **Trading** takes place **periodically** between $t = 1$ and T.[4] At T, the value of the risky asset[5] is disclosed to all market participants, who thus become equally informed.[6]

2. The agents trade **one risky asset**. Its value V_t is an indicator of the current economic situation, and is supposed to exhibit either a positive, or a negative evolution from one trade moment to the other. Consequently, V_t is modelled as a binary random variable with two possible values: high and low $V_t \in \{V_H, V_L\}$.[7] For simplicity, we further take $V_H = 1$ and $V_L = 0$.[8]

3. There are **no budget constraints**.[9]

4. There are three homogenous[10] non-overlapping groups of investors $g \in \{II, PI, N\}$ trading in the market, namely:

   - the *perfectly informed investors (PI)*[11],
   - the *users of practical decision rules (II)*[12] and
   - the *noise traders (N)*;
   - in addition there is only one competitive **market maker** *(MM)*, with the principal task of periodical price setting, in order to maintain a fair, orderly, liquid and efficient market.[13]

   The *probability that an investor belongs to a certain category*, conditional on a high value of the risky asset and on the common information[14] at t, equals, in the view of the market maker, the *constant* proportion of that category to the totality of investors: $P(g|V_t = V_H, h_{t-1}) = p_g = n_g/n$,

[4]We model the trade time discreetly. Easley, Kiefer, O'Hara and Paperman (1996) analyze the continuous case, with uninformed orders arriving stochastically at a constant exogenous intensity.

[5]See Assumption (2).

[6]We can assume that the whole trading period consists of many uninterrupted episodes of the form $t = \overline{1, T}$. These correspond in practice to the trading days, months or years, naturally separated by intervening periods, which can be further decomposed into smaller units. At the beginning ($t = 0$) and at the end ($t = T$) of such an interval the agents dispose of homogenous information. In Glosten and Milgrom (1985) (pp. 76-77) the trading also takes place between $t = 1$ and T_0. At T_0 the value of the risky asset is publicly announced.

[7]Glosten and Milgrom (1985) (p. 71) also allow for two possible values of the risky asset, but their definition differs from the one given here.

[8]The benchmark value $V_L = (E[V_t|\text{bad economic situation}]) = 0$ emphasizes the fact, that during a negative economic development the risky asset is as good as worthless. In turn, V_H can take any positive value. E.g. if the risky asset yields a constant dividend D and the market participants consider its economic life-time to be infinite, then V_H equals the time series of the discounted dividends: $V_H = (E[V_t|\text{good economic situation}]) = \sum_{i=1}^{\infty} \frac{D}{(1+r)^i} = \frac{D}{r}$, with r being the risk-free interest rate.

[9]According to this assumption, the financial situation of the agents does not represent a restriction for the formulation of their actions and hence for the periodical price calculation.

[10]The notion "homogenous" refers here to informational homogeneity: all members of a group dispose at the same time of the same information and interpret it in the same way.

[11]If we assume that private information plays a more important role in decision making than method-based information, this first group of investors can be viewed as consisting of privately informed traders. In turn, if we bear in mind the significance of herd behavior for choosing the proper action in a given economic situation, the best informed traders are those who take account of the price movements caused by the conjoint action of other agents.

[12]This group can include both fundamental and technical analysts, as users of systematical practical methods for making their decisions. The former concentrate on the calculus of different global, market or firm-specific indicators, while the latter study the evolution of past prices and transaction volume.

[13]In financial market terminology there is no clear and unitary assignment of the attributes mentioned here to a certain market participant. At the NYSE or AMEX they are entirely undertaken by the so called specialists, while the market makers act more as brokers/dealers. On the floor of the Frankfurt Stock Exchange the so called Amtlicher Kursmakler is responsible for the price setting and maintaining of proper market conditions. In our view, the name "market maker" best reflects the principal task of such an agent, namely "to make the market" (i.e. to bring together the investors' demand and supply).

[14]For the definition of common information see Assumption (7).

so that $p_{PI} + p_{II} + p_N = 1$.[15] Furthermore, there are always some noise traders trading in the market: $p_N > 0$.[16]

5. All market participants are **risk-neutral** (i.e. their utility functions exhibit a linear evolution subject to their wealth)[17] and **myopic** (i.e. their trading horizon is restricted to one period).[18]

6. We make the following assumptions regarding the **rationality** of the agents' behavior:[19]

   - the *perfectly informed investors* act *fully rationally* (i.e. they form accurate expectations about the economic situation on the basis of the available information);
   - the *users of practical decision rules* behave *not fully rationally* (i.e. their decisions are based upon their method-specific information signals, as long as these signals are in accordance with the state of the economy at large [20]; otherwise their information is of no further relevance for their actions, and they trade randomly[21]);[22]
   - the *noise traders* are, by contrast, *irrational* (because they do not act on new information and accordingly form periodical expectations based only on past data.[23]
   - the *market maker* reacts *passively* to the orders of the investors;[24] she uses the Bayes rule to formulate her assessments of the economic situation and thus to set prices.[25]

7. At every time t the market participants dispose of **common information** $h_{t-1} \equiv \{x_i \vee X_i; i = 1, \bar{t} - 1\}$ (with $h_0 = \emptyset$), consisting of the sequence of past actions (x_i) and of past prices (X_i)[26].

8. According to their **degree of information**, the agents are grouped in two distinct categories:

[15]For analogous assumptions concerning a constant proportion of the informed investors see Copeland and Galai (1983) (p. 1459) or Easley/O'Hara (1987) (p. 71-72). Easley, Kiefer and O'Hara (1997) (p. 821) estimate the fraction of the informed investors in the US-market at about 17%. A ZEW survey (see Rebitzky (ZEW-2004), p. 3) points out the fraction of the technical analysts in the German Foreign Exchange Market, to be approximatively 30%, while the share of fundamentalists participating in trading is roughly 60%.

[16]As long as the number of active noise traders remains strictly positive, the market maker can compensate the losses from doing business with informed agents by the gains from transactions with the uninformed ones. Otherwise, the excessive and repeated losses resulting from buying or selling exclusively from or to the informed traders could cause a definitive trade cessation. See Easley and O'Hara (1987) (p. 72).

[17]The risk-neutrality of the market maker is explicitly assumed by Glosten and Milgrom (1985) (p. 77) or Easley and O'Hara (1987) (p. 71).

[18]The term of "myopia" is used here in the sense of short-term behavior. The agents do not account for the effects of their actions on the prices of subsequent periods. Hence, their periodical trading decisions are independent. See O'Hara (1995) (p. 132 and 157). The survey by Taylor and Allen (1992) (pp. 308-309) make the case for an appropriate application of practical decision rules in a market with myopic investors. The practical use of technical trading rules is ascertained to be greater for short-, than for long-term decisions. Brown and Jennings (1990) (p. 534) demonstrate furthermore, that the technical analysis exhibits a certain value in a linear two-period rational expectations equilibrium of a market with myopic investors.

[19]Glosten and Milgrom (1985) (p. 77) assume that all investors are fully rational, i.e. they are expected utility-maximizers. Uninformed investors, who trade at random, can be found in Kyle (1985) (p. 1315), Easley, Kiefer, O'Hara and Paperman (1996) (pp. 1408-1409) or Grammig, Schiereck and Theissen (2000) (p. 622).

[20]Information is not in accord with the economic situation in one of the following situations: either the signal is one and the value of the risky asset low, or the signal is zero and the value high.

[21]Random trading refers to equal buy and sell probabilities.

[22]In fact, the users of practical decision rules estimate the current economic situation with the aid of the Bayes rule. See Section 3.3. Even so, their behavior is not fully rational, because they do not always use the entire available information in order to make their decisions.

[23]The trade reasons of these investors can be seen as exogenous (e.g. liquidity needs).

[24]According to Harris (2003) (pp. 278-279), market makers indeed behave in reality as passive traders, unable to control the timing of their trades. Madhavan (2000) (p. 212) assumes a similar behavior.

[25]An analogous assumption is to be found in Easley and O'Hara (1987) (pp. 75-77).

[26]For the respective definitions of actions and prices see the Assumptions (9) respectively (8).

informed and the *uninformed*.[27] The perfectly informed investors and the users of practical decision rules belong to the first group and derive periodically either a zero or a negative information signal $s_t \in \{0, 1\}$.[28] (It is also assumed that the informed investors receive an informative signal at every trading moment.) The noise traders and the market maker are uninformed.[29]

- The *perfectly informed traders* deliver accurate forecasts, because they receive a positive signal, when the value of the risky asset is high V_H, and a zero-signal, in case of a low value V_L , namely:[30]

$$\begin{aligned} P(s_{PIt} = 1|V_t = V_H, h_{t-1}) &= 1 - P(s_{PIt} = 0|V_t = V_H, h_{t-1}) \equiv 1 \\ P(s_{PIt} = 1|V_t = V_L, h_{t-1}) &= 1 - P(s_{IIt} = 0|V_t = V_L, h_{t-1}) \equiv 0. \end{aligned} \tag{1}$$

- The *users of practical decision rules* receive positive information at time t[31], given the value of the risky asset, with the following probabilities:[32]

$$\begin{aligned} q_{HIIt} \equiv P(s_{IIt} = 1|V_t = V_H, h_{t-1}) &= 1 - P(s_{IIt} = 0|V_t = V_H, h_{t-1}) \\ q_{LIIt} \equiv P(s_{IIt} = 1|V_t = V_L, h_{t-1}) &= 1 - P(s_{IIt} = 0|V_t = V_L, h_{t-1}). \end{aligned} \tag{2}$$

[27] In this context, relevant information can be obtained by the investors as a result of systematical analysis of the market data. Consequently, the periodical information of the market maker about the entire order flow cannot be viewed as relevant. It can, in any case, be used by the market maker for ascertaining price-relevant parameters, as can be seen in Section 4.2, footnote 61. In this case, the market maker herself disposes of the entire information available and acts according to our view of rationality in a fully rational manner.

[28] We designate below $s_t = 1$ as a "positive signal" and $s_t = 0$ as a "zero-signal".

[29] Most models act on the assumption of the existence of two groups of investors: the informed (often called "sophisticated investors" or "rational traders") and the uninformed (mostly designated as "liquidity traders" or "noise traders"). The fundamentalists are mostly integrated into the first group, while the technical analysts are viewed as being more uninformed. See De Long, Shleifer, Summers and Waldmann (1990) (p. 706). In real market settings, Harris (2003) (pp. 226-235) distinguishes among four categories of informed traders, i.e. "value traders" (who use the entire available information for ascertaining the fundamental value of the asset), "news traders" (who only act upon new information), "information-oriented technical analysts" (who profit by identifying predictable price patterns) and "arbitrageurs" (who speculate on instruments, which are inconsistently priced relative to each other).

[30] In the reality perfectly informed investors are rarely to be met. Here, such an assumption serves the purpose of simplifying the model structure and emphasizing the existence of an better informed group of investors (being thus viewed as benchmark).

[31] In contrast to other market microstructure models, the users of practical decision rules (i.e. the technical or fundamental analysts) are viewed here as informed traders, the reason being that these traders systematically derive new information, availing of their own method. This would not be the case for the noise traders, who only dispose of the publicly available information and make no efforts to obtain more information. However, we make no assumption about the precision of the information of the users of practical decision rules and correspondingly about its capability to generate appropriate actions. (In fact, we assume and model only the *possibility* of this information having some precision.) This should be reflected by the prices and misperceptions (see Section 4).

In sustaining the assumption that practical decision rules (such as those delivered by the use of fundamental or technical instruments) could offer valuable information, we can make some remarks. On the one hand, the fundamentalists are usually supposed to gather useful information during the determination process of the so called fundamental value of the risky asset. (This value represents an intrinsic value which is independent of the transitory market phenomena, being thus often utilized as reference for the evaluation of price movements.) On the other hand, several approaches support the relevance of technical information (as a combination of the past prices and volumes) or of some technical methods (such as moving average) for obtaining positive excess returns in the stock and foreign exchange market. (Treynor and Ferguson (1985), Brown and Jennings (1989) or Blume, Easley and O'Hara (1994) account from a theoretical point of view for the rationality of the technical analysis and the effectiveness of using a price sequence instead of single prices. They also emphasize the benefits of the combination of past prices and volumes for choosing optimal trading strategies. Using real market data, Brock, Lakonishok and LeBaron (1992), Allen and Karjalainen (1995), Neely, Weller and Dittmar (1997) or Lo, Mamaysky and Wang (2000) evaluate the success of different technical methods as basically positive.

[32] In Easley and O'Hara (1987) (p. 71) the signals of the market participants are informative with a given probability. We assume that the information signals of the fundamental analysts rely on the fundamental value of the risky asset X_t^f, derived from the analysis of the global, market and firm-specific situation (see Murphy (2000), p. 24), so that: $q_{HPIt} = f(X_{-\infty}, ..., X_{t-1}, X_{-\infty}^f, ..., X_{t-1}^f, h_{t-1})$ (and analogous for q_{LPIt}). The technical analysis extracts information from the past prices and volume (see Murphy (2000), p. 165), so that: $q_{HIIt} = f(X_{-\infty}, ..., X_{t-1}, U_{-\infty}, ..., U_{t-1}, h_{t-1})$ (and analogous for q_{LIIt}), with U_t the periodical volume (which further remains unconsidered).

- The *noise traders* trade at random (i.e. with equal buy and sell probabilities in each economic situation).[33]

- The *market maker* accommodates the buy and sell orders of the investors and executes them at the quoted buy prices (**ask** X_{Bt}) and sell prices (**bid** X_{St}).[34] The prices are *competitively* set[35], so that the market maker gains no profit from any of the buys and sells undertaken.[36]

  The ask price normally exceeds the bid price by the amount of the so called **bid-ask spread**. The spread should cover the order processing costs, the inventory costs and the costs of adverse selection, that arise during trading[37], and represents the only source of earnings for the market maker. We expect a *constant* fraction of the spread to be responsible for the first two cost elements. We also require that all transactions be processed by the market maker (i.e. there is *no direct trade* between the investors).[38]

9. In order to characterize the **trading alternatives** of the different agents, we introduce the following notations for the buy probability, conditional upon a high and a low value of the risky asset, respectively:

$$\begin{aligned} u_{gt} &= P(x_{gt} = B|V_t = V_H, h_{t-1}) &= 1 - P(x_{gt} = S|V_t = V_H, h_{t-1}) \\ v_{gt} &= P(x_{gt} = B|V_t = V_L, h_{t-1}) &= 1 - P(x_{gt} = S|V_t = V_L, h_{t-1}), \end{aligned} \tag{3}$$

with $g \in \{II, PI, N, MM\}$.

In every period *all market participants trade once* and only *one unit* of the risky asset.[39] Moreover, they trade *actively*[40], by *either buying or selling* the asset[41], i.e. $x_{gt} \in \{B, S\}$ (with $g \in \{II, PI, N, MM\}$).

- The *perfectly informed investors* obtain a fully informative signal at every trade time. This fact allows them to exactly recognize the economic situation and therefore to choose the appropriate trade alternative.

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[33] For an analogous assumption see Kyle (1985) (p. 1315).

[34] We assume, that all transactions take place at exactly the quoted prices. In reality, the dealers can treat some investors preferentially, by offering them price reductions. See Harris (2003) (p. 281).

[35] The monopolistic power of the single market maker is constrained by her duty to set fair and efficient prices. This analysis can therefore be viewed as a marginal case of the general situation with many competing market makers. Some real markets (i.e. NYSE) do indeed function with only one market maker per traded asset. Even in such market settings there are factors accounting for competition, such as: competing markets with lower bid-ask spreads, limit orders, other specialists, floor traders, etc. See Demsetz (1968) (p. 43-44) or Harris (2003) (p. 298).

[36] The competitive price setting represents an application of the zero expected profit-condition. Accordingly, the prices equal the expectations of the market maker regarding the value of the risky asset, conditional upon the available information. See Copeland and Galai (1983) (p. 1462) and O'Hara (1995) (p. 146). Glosten and Milgrom (1985) (p. 79-81) and Easley and O'Hara (1987) (p. 74) stress the zero expected profit-condition as well.

[37] See Stoll (1989) (p. 115) or Grammig, Schiereck and Theissen (2000) (p. 20-21).

[38] The modelled market functions hence as a quote-driven system, as defined in DeMarchi and Foucault (1998) (p. 36).

[39] The restrictions upon the order size and the trading frequency, respectively, should help avoid a premature trade cessation. Otherwise, the perfectly informed traders could already trade at $t = 1$ as much as possible, thereby making the price from the beginning fully informative and eliminating every trade incentive. While Glosten and Milgrom (1985) (p. 76) also assume a one-unit order size, Easley and O'Hara (1987) (p. 72) allow for its discreet variation. In turn, Kyle (1985) (p. 1317) models the order size (in an sequential auction-equilibrium) as a continuous normally distributed random variable.

[40] In contrast to Glosten and Milgrom (1985) (p. 76) or Easley, Kiefer, O'Hara and Paperman (1986) (p. 1409) we do not account for "hold" as a third possible action, besides "buy" and "sell". This is due to several assumptions which create the premises for an active trade at every time, i.e. that both groups of informed investors periodically receive new information, the noise traders are always active by trading at random and the market maker is supposed to accommodate the orders of the agents.

[41] The buys and sells are thus independent.

- The *users of practical decision rules* buy, if their method-based signal is positive and the economic situation good (i.e. the value of the risky asset is high), and sell if they receive a zero-signal and the economic evolution is bad (i.e. the value of the risky asset is low). Otherwise, they trade at random, either buying or selling with the same probability (1/2).
- Because the *noise traders* do not dispose of new information, they trade in every period at random, i.e. they buy or sell with the same probability (1/2), independent of the state of the economy, such as: $u_{Nt} = v_{Nt} = 1/2$.
- The *market maker* calculates in every trade period the ask and bid prices on the basis of her assessments regarding the economic situation. She is subsequently committed to fulfill all the received investors' orders at these prices.

Moreover, the investors are assumed to *simultaneously* hand on their orders to the market maker.[42] Hence, they get no information about the contemporaneous actions of other agents.[43] The prices are determined only after the orders have been placed (namely on this basis) (see Section 3.1).[44] Moreover, the investors remain *anonym*.[45]

10. At the time t all market participants are acquainted with the **a-priori assessment** $p_{t-1} = P(V_t = V_H|h_{t-1})$, formulated on the basis of the common (i.e. past) information.[46]

3 The trading process

In this Section we discuss the trading steps and derive intermediary results important for the price formation described in Section 4.

3.1 The structure of the trading process

In every trade period, the market participants perform the following two trading steps:[47]

- the *investors*:

[42] An analogous assumption is made by Kyle (1985) (p. 1315). Glosten and Milgrom (1985) (p. 78), Easley and O'Hara (1987) (p. 73) or Easley, Kiefer and O'Hara (1997) (p. 811) model the trade sequentially, in a given (probabilistically) succession of the investors. Back and Baruch (2003) (p. 447-448) demonstrate the equivalence of the Glosten and Milgrom's sequential equilibrium with Kyle's continous equilibrium, for small order sizes and frequently uninformed transactions.

[43] In our approach, the investors cannot directly observe the actions of other market participants and are therefore unable to form an opinion about the contemporaneous information of these other traders. Consequently, the learning takes place exclusively on the basis of the collective experience and, in addition for the informed traders, by means of new information. Easley, Kiefer, O'Hara and Paperman (1996) and Easley, Kiefer and O'Hara (1997) analyze the case with continuously arriving uninformed orders, whereby the arrival succession is modelled as a stochastic Poisson-process.

[44] The contemporaneous prices thus remain unknown to the investors in the moment of choosing their actions. (The trading rules rely consequently only on the available information and not on other price-based rules such as profit maximization.) Because the single prices are not solely relevant, the analysis of the informative content of a price sequence (which is possible in a context with strategic traders) could represent an interesting avenue of further research.

[45] The anonymity of investors and market transparency have been intensively studied in the attendant literature. See the summaries of O'Hara (1995) (pp. 252-268) and Madhavan (2000) (pp. 236-241). The general conclusion stresses a less acute adverse selection in transparent market settings, which determines lower bid-ask spreads, at least for some trader categories.

[46] The a-priori assessment relies on the common information of the market at a given time and thus reflects the public opinion. The moment $t = 0$ characterizes the original state before the beginning of trade. At that given time, all traders are aware of the a-priori probability $p_0 = P(V_1 = V_H) = 1 - P(V_1 = V_L)$ of a high risky value at the begin of that trade interval. This probability is influenced by the results of the previous trade intervals. (If the value of the asset was high at the end of the previous interval, then $V_T = V_H$, p_0 increases, otherwise it decreases.) During the coming trade interval investors gather decision-relevant information (on the basis of the collective experience or also with the help of new periodical information.

[47] Our trading structure is similar to Kyle (1985) (p. 1315-1316). By contrast, the investors act on the basis of previously fixed prices in Easley, Kiefer and O'Hara (1997) (p. 811).

- ◦ determine the optimal action x_{gt} (with $g \in \{II, PI, N\}$), on the basis of the common and, contingently, on the new information (whereby the latter applies only to the informed traders) (see Section 3.2) and release the orders;
  - ◦ update their assessments of the value of the risky asset $P_{aktual,gt}$ (with $g \in \{II, PI, N\}$) (see Section 3.3);

- the *market maker*:
  - ◦ updates her assessments with respect to the economic situation P_{BMMt} and P_{SMMt} (see Section 3.3);
  - ◦ ascertains the prices X_{Bt} and X_{St} (see Section 4.1);
  - ◦ executes the orders of the investors;

- following (that means after the price calculation and disclosing), the investors can assess their expected profits G_{gt} with the aid of $P_{aktual,gt}$ (with $g \in \{II, PI, N\}$) and the prices X_{Bt}, X_{St} (see Section 3.4).

The price formation is based on the market makers' assessments concerning the economic situation during the respective trade period. These assessments rely on the probabilities the market maker ascertains in the case of investors wanting to buy or to sell. Furthermore, the type of orders released depends on the the group-specific decision rules. Accordingly, we differentiate several steps of price formation, which will be presented in the following Sections 3.2, 3.3 and 3.4. In fact, only the assessments, actions and profit expectations of the market maker are relevant for the price derivation. Actually, the corresponding considerations and actions of the investors are responsible for the reactions of the market maker. The following sections discuss the behavior of all market participants as a consequence.

Our main goal is to ascertain the price formulas subject to the variables of interest, namely q_{HIIt} respectively q_{LIIt} (as an expression of the precision of the method-based information) and p_{II} (the fraction of the users of practical decision rules). Subsequently (in Section 4.2), we analyze the price movements due to the c.p. variation of these factors.

3.2 The actions of the market participants

As we have already assumed under Hypothesis (9), each group of investors shapes its actions according to different trading rules. From the viewpoint of the market maker, these rules can be summarized as follows:

- the *perfectly informed investors* always buy if they receive positive information (which clearly signalizes a high value of the risky asset) and sell when the information is negative (because in this case the risky value will certainly be low).

$$x_{PIt} = \begin{cases} B & \text{, if} \quad s_{PIt} = 1 \quad (\equiv \quad V_t = V_H) \\ S & \text{, if} \quad s_{PIt} = 0 \quad (\equiv \quad V_t = V_L). \end{cases} \tag{4}$$

Hence, the buy and sell probabilities in a good or a bad economic situation are computed as follows:

$$\begin{aligned} \mathbf{u_{PIt}} &= P(x_{PIt} = B | V_t = V_H, h_{t-1}) &= P(x_{PIt} = B | s_{PIt} = 1, h_{t-1}) &\equiv \mathbf{1} \\ \mathbf{v_{PIt}} &= P(x_{PIt} = B | V_t = V_L, h_{t-1}) &= 1 - P(x_{PIt} = S | s_{PIt} = 0, h_{t-1}) &\equiv \mathbf{0}. \end{aligned} \tag{5}$$

- the *users of practical decision rules* buy if the value of the risky asset is high and if they receive a positive information signal, sell for a zero-signal and a bad state of the economy at large, and act at random otherwise.

$$x_{IIt} = \begin{cases} B & \text{, if} \quad s_{IIt} = 1 \quad \text{and} \quad V_t = V_H \\ S & \text{, if} \quad s_{IIt} = 0 \quad \text{and} \quad V_t = V_L \\ B \vee S & \text{, otherwise.} \end{cases} \tag{6}$$

Consequently, we have:

$$\begin{aligned} P(x_{IIt} = B | s_{IIt} = 1, V_t = V_H, h_{t-1}) &= 1; \\ P(x_{IIt} = S | s_{IIt} = 1, V_t = V_L, h_{t-1}) &= 1. \end{aligned}$$

With the following notations:

$$\begin{aligned} P(x_{IIt} = B | s_{IIt} = 1, V_t = V_L, h_{t-1}) &\equiv r_{L1IIt} \\ P(x_{IIt} = B | s_{IIt} = 0, V_t = V_H, h_{t-1}) &\equiv r_{H0IIt}, \end{aligned} \tag{7}$$

the probabilities u_{IIt} and v_{IIt} satisfy:

$$\begin{aligned} u_{IIt} &= P(x_{IIt} = B | V_t = V_H, h_{t-1}) \\ &= P(x_{IIt} = B | s_{IIt} = 1, V_t = V_H, h_{t-1}) \cdot P(s_{IIt} = 1 | V_t = V_H, h_{t-1}) \\ &+ P(x_{IIt} = B | s_{IIt} = 0, V_t = V_H, h_{t-1}) \cdot P(s_{IIt} = 0 | V_t = V_H, h_{t-1}) \\ &= q_{HIIt} + r_{H0IIt} \cdot (1 - q_{HIIt}); \\ v_{IIt} &= P(x_{IIt} = B | V_t = V_L, h_{t-1}) \\ &= P(x_{IIt} = B | s_{IIt} = 1, V_t = V_L, h_{t-1}) \cdot P(s_{IIt} = 1 | V_t = V_L, h_{t-1}) \\ &+ P(x_{IIt} = B | s_{IIt} = 0, V_t = V_L, h_{t-1}) \cdot P(s_{IIt} = 0 | V_t = V_L, h_{t-1}) \\ &= r_{L1IIt} \cdot q_{LIIt}. \end{aligned}$$

According to Assumption (9), $r_{H0IIt} = r_{L1IIt} = 1/2$. This results in:

$$\begin{aligned} \mathbf{u_{IIt}} &= \frac{\mathbf{1 + q_{HIIt}}}{\mathbf{2}} \\ \mathbf{v_{IIt}} &= \frac{\mathbf{q_{LIIt}}}{\mathbf{2}}. \end{aligned} \tag{8}$$

The buy (u_{IIt}) and sell ($1 - v_{IIt}$) probabilities during a good and bad economic situation, respectively, depend linearly on the corresponding probabilities of a positive (q_{HIIt}) and a zero information (q_{LIIt}), respectively.

- *noise traders* are "blind" traders who always undertake an action at the prices set by the market maker. Hence we have:

$$x_{Nt} = \begin{cases} B & \text{, with} \quad P(x_{Nt} = B | h_{t-1}) \\ S & \text{, with} \quad P(x_{Nt} = S | h_{t-1}). \end{cases} \tag{9}$$

According to Assumption (9), they act at random. This results in:

$$\mathbf{u_{Nt}} \quad = \quad \mathbf{v_{Nt}} \quad = \quad \frac{\mathbf{1}}{\mathbf{2}}. \tag{10}$$

The noise traders consequently buy or sell with the same probability, independent of the economic situation:

$$\begin{aligned}
P(x_{Nt} = B|h_{t-1}) &= P(x_{Nt} = B|V_t = V_H, h_{t-1}) \cdot P(V_t = V_H|h_{t-1}) \\
&+ P(x_{Nt} = B|V_t = V_L, h_{t-1}) \cdot P(V_t = V_L|h_{t-1}) \\
&= u_{Nt} \cdot P(V_t = V_H|h_{t-1}) + v_{Nt} \cdot [1 - P(V_t = V_H|h_{t-1})] \\
&= 1/2 \cdot p_{t-1} + 1/2 \cdot [1 - p_{t-1}] \ = \ 1/2 \\
P(x_{Nt} = S|h_{t-1}) &= P(x_{Nt} = S|V_t = V_H, h_{t-1}) \cdot P(V_t = V_H|h_{t-1}) \\
&+ P(x_{Nt} = S|V_t = V_L, h_{t-1}) \cdot P(V_t = V_L|h_{t-1}) \\
&= (1 - u_{Nt}) \cdot p_{t-1} + (1 - v_{Nt}) \cdot [1 - p_{t-1}] \ = \ 1/2.
\end{aligned}$$

- the *market maker* accommodates the investors' orders and engages in their execution at two different prices: the ask (which has to be payed by a buying investor) and the sell (for an investor wanting to sell). Hence, she assesses the probabilities u_{MMt} and v_{MMt}, subject to the investors' readiness to buy in a given economic situation. From the market maker's viewpoint u_{MMt} and $1 - v_{MMt}$ represent the probabilities that buys ($x_t = B$) and sells ($x_t = S$), respectively, take place[48], taking into account the economic situation and the common information at the time of order assignment.[49]

$$\begin{aligned}
u_{MMt} &= P(x_t = B|V_t = V_H, h_{t-1}) \\
&= P(x_t = B|V_t = V_H, g = II, h_{t-1}) \cdot P(g = II|V_t = V_H, h_{t-1}) \\
&+ P(x_t = B|V_t = V_H, g = PI, h_{t-1}) \cdot P(g = PI|V_t = V_H, h_{t-1}) \\
&+ P(x_t = B|V_t = V_H, g = N, h_{t-1}) \cdot P(g = II|V_t = V_H, h_{t-1}) \\
&= u_{IIt} \cdot p_{II} + u_{PIt} \cdot p_{PI} + u_{Nt} \cdot p_N \ = \ u_{IIt} \cdot p_{II} + p_{PI} + u_{Nt} \cdot (1 - p_{PI} - p_{II}) \\
&= (u_{IIt} - 1/2) \cdot p_{II} + 1/2 \cdot p_{PI} + 1/2 \\
v_{MMt} &= P(x_t = B|V_t = V_L, h_{t-1}) \ = \ v_{IIt} \cdot p_{II} + v_{PIt} \cdot p_{PI} + v_{Nt} \cdot p_N \\
&= v_{IIt} \cdot p_{II} + v_{Nt} \cdot (1 - p_{PI} - p_{II}) \ = \ (v_{IIt} - 1/2) \cdot p_{II} - 1/2 \cdot p_{PI} + 1/2.
\end{aligned}$$

The market maker, therefore, calculates the buy and sell probabilities u_{MMt} and v_{MMt}, respectively, as weighted sums of the similar probabilities for each group of investors u_{gt} and v_{gt}, respectively (with $g \in \{PI, II, N\}$). The weights are the particular proportions of each group to the totality of investors p_g. The independence of the investors' buys and sells[50] ensures the independence of the market maker's buys and sells: $P(x_t = S|V_t = V_H, h_{t-1}) = 1 - u_{MMt}$ and $P(x_t = S|V_t = V_L, h_{t-1}) = 1 - v_{MMt}$.

[48] The notation is always made from the investors' viewpoint. The market maker then executes the contrary action (i.e. sell for $x_t = B$ and buy for $x_t = S$).

[49] $x_t = B$ and $x_t = S$ do not therefore mean, that one certain investor buys and sells, respectively, but the possibility that the market maker receives at t buy and sell orders, respectively.

[50] See Assumption (9), footnote 41.

Considering the simplifying Assumption (9), we derive the buy probabilities as non-linear expressions of the variables of interest q_{HIIt}, q_{LIIt} and p_{II}:

$$\begin{aligned}
\mathbf{u_{MMt}} &= \frac{\mathbf{q_{HIIt} \cdot p_{II} + p_{PI} + 1}}{\mathbf{2}} \\
\mathbf{v_{MMt}} &= \frac{\mathbf{(q_{LIIt} - 1) \cdot p_{II} - p_{PI} + 1}}{\mathbf{2}}.
\end{aligned} \tag{11}$$

While both u_{MMt} and v_{MMt} remain linearly dependent on p_{PI}, which is in fact a result of the precision of the perfect information, a joint effect of the qualitative and quantitative factors appears with regard to the users of practical decision rules.

3.3 Assessments of the market participants

In every trade period the agents form group-specific assessments of the risky value and update them on the basis of the previous actions and prices, and also (only for the informed traders) of the new information.[51] Only the market maker's assessments are of immediate interest for the price derivation. In any case, we also derive the assessments of the investors, which further allow the calculation of their profits and of the misperceptions (see Sections 3.4, 4.1 and 4.3).

- The *perfectly informed investors* provide perfect assessments of the value of the risky asset, by means of the Bayes rule:

$$\begin{aligned}
P_{1PIt} &= P(V_t = V_H | s_{PIt} = 1, h_{t-1}) \\
&= \frac{P(s_{PIt} = 1 | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(s_{PIt} = 1 | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1}) + P(s_{PIt} = 1 | V_t = V_L, h_{t-1}) \cdot P(V_t = V_L | h_{t-1})} \\
&= \frac{1 \cdot p_{t-1}}{1 \cdot p_{t-1} + 0 \cdot (1 - p_{t-1})} \\
P_{0PIt} &= P(V_t = V_H | s_{PIt} = 0, h_{t-1}) \\
&= \frac{P(s_{PIt} = 0 | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(s_{PIt} = 0 | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1}) + P(s_{PIt} = 0 | V_t = V_L, h_{t-1}) \cdot P(V_t = V_L | h_{t-1})} \\
&= \frac{0 \cdot p_{t-1}}{0 \cdot p_{t-1} + 1 \cdot (1 - p_{t-1})}.
\end{aligned}$$

  Consequently we obtain:

$$\begin{aligned}
\mathbf{P_{1PIt}} &= \mathbf{1} \\
\mathbf{P_{0PIt}} &= \mathbf{0}.
\end{aligned} \tag{12}$$

- The *users of practical decision rules* assess the value of the risky asset, also with the aid of the Bayes rule:

$$\begin{aligned}
P_{1IIt} &= P(V_t = V_H | s_{IIt} = 1, h_{t-1}) = \frac{P(s_{IIt} = 1 | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(s_{IIt} = 1 | h_{t-1})} \\
P_{0IIt} &= P(V_t = V_H | s_{IIt} = 0, h_{t-1}) = \frac{P(s_{IIt} = 0 | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(s_{IIt} = 0 | h_{t-1})}.
\end{aligned}$$

[51] These assessments could be just as well understood as beliefs or expectations about the economic situation at time t. They reflect both past and new information and should be thus understood as a-posteriori probabilities, which mostly differ from a-priori assessments. We prefer the term "assessment" due to the fact that we work with probabilities and not with random variables.

Consequently, we obtain:

$$\begin{aligned} \mathbf{P_{1IIt}} &= \frac{\mathbf{q_{HIIt} \cdot p_{t-1}}}{\mathbf{q_{HIIt} \cdot p_{t-1} + q_{LIIt} \cdot (1 - p_{t-1})}} \\ \mathbf{P_{0IIt}} &= \frac{\mathbf{(1 - q_{HIIt}) \cdot p_{t-1}}}{\mathbf{(1 - q_{HIIt}) \cdot p_{t-1} + (1 - q_{LIIt}) \cdot (1 - p_{t-1})}}. \end{aligned} \tag{13}$$

This result points out the non-linear dependence of the method-based assessments of the informational precision (q_{HIIt} and q_{LIIt}) and of the a-priori assessment (p_{t-1}).

- The assessments of the *noise traders* regarding the economic situation rely exclusively upon the common information at t:

$$\mathbf{P_{Nt}} = P(V_t = V_H | h_{t-1}) = \mathbf{p_{t-1}}. \tag{14}$$

- The *market maker* assesses the economic situation starting from the aggregate investors' demand. She does not necessarily include the number of received buy or sell orders in her calculations[52], but ascertains the probability that such orders are carried out.

$$\begin{aligned} P_{BMMt} &= P(V_t = V_H | x_t = B, h_{t-1}) = \frac{P(x_t = B | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(x_t = B | h_{t-1})}, \\ &\quad \text{if the investors buy from the market maker} \\ P_{SMMt} &= P(V_t = V_H | x_t = S, h_{t-1}) = \frac{P(x_t = S | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(x_t = S | h_{t-1})}, \\ &\quad \text{if the investors sell to the market maker.} \end{aligned}$$

We further obtain:

$$\begin{aligned} \mathbf{P_{BMMt}} &= \frac{\mathbf{u_{MMt} \cdot p_{t-1}}}{\mathbf{u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1 - p_{t-1})}} \\ \mathbf{P_{SMMt}} &= \frac{\mathbf{(1 - u_{MMt}) \cdot p_{t-1}}}{\mathbf{(1 - u_{MMt}) \cdot p_{t-1} + (1 - v_{MMt}) \cdot (1 - p_{t-1})}}. \end{aligned} \tag{15}$$

The assessments of the market maker regarding the economic situation constitute a combination of past information (as reflected in the a-priori assessment p_{t-1}) and the probabilities of the contemporaneous actions of the investors (as illustrated by the probabilities u_{MMt} and v_{MMt}).

It would be desirable to represent the assessments of the market maker as functions of the variables of interest: q_{HIIt}, q_{LIIt} and p_{II}. Therefore, we introduce the formulas (11) derived in Section 3.2 for u_{MMt} and v_{MMt}:

$$\begin{aligned} P_{BMMt} &= \frac{[u_{IIt} \cdot p_{II} + p_{PI} + u_{Nt} \cdot p_N] \cdot p_{t-1}}{[u_{IIt} \cdot p_{II} + p_{PI} + u_{Nt} \cdot p_N] \cdot p_{t-1} + [v_{IIt} \cdot p_{II} + v_{Nt} \cdot p_N] \cdot (1 - p_{t-1})} \\ P_{SMMt} &= \frac{[(1 - u_{IIt}) \cdot p_{II} + (1 - u_{Nt}) \cdot p_N] \cdot p_{t-1}}{[(1 - u_{IIt}) \cdot p_{II} + (1 - u_{Nt}) \cdot p_N] \cdot p_{t-1} + [(1 - v_{IIt}) \cdot p_{II} + p_{PI} + (1 - v_{Nt}) \cdot p_N] \cdot (1 - p_{t-1})}. \end{aligned}$$

Under Assumption (9) (that $u_{Nt} = v_{Nt} = 1/2$) the calculation simplifies to:

$$\begin{aligned} P_{BMMt} &= \frac{[2u_{IIt} \cdot p_{II} + 2p_{PI} + p_N] \cdot p_{t-1}}{[2u_{IIt} \cdot p_{II} + 2p_{PI} + p_N] \cdot p_{t-1} + [2v_{IIt} \cdot p_{II} + p_N] \cdot (1 - p_{t-1})} \\ P_{SMMt} &= \frac{[2(1 - u_{IIt}) \cdot p_{II} + p_N] \cdot p_{t-1}}{[2(1 - u_{IIt}) \cdot p_{II} + p_N] \cdot p_{t-1} + [2(1 - v_{IIt}) \cdot p_{II} + 2p_{PI} + p_N] \cdot (1 - p_{t-1})}. \end{aligned}$$

[52] This would be possible using the proportion of the contemporaneous buy and sell orders (see Section 4.2, footnote 61). The market maker could thus estimate the a-priori probability p_{t-1}.

But $p_{II} + p_{PI} + p_N = 1$, so that:

$$\begin{aligned}
P_{BMMt} &= \frac{[(2u_{IIt}-1)\cdot p_{II} + p_{PI} + 1]\cdot p_{t-1}}{[(2u_{IIt}-1)\cdot p_{II} + p_{PI} + 1]\cdot p_{t-1} + [(2v_{IIt}-1)\cdot p_{II} - p_{PI} + 1]\cdot(1-p_{t-1})}\\
P_{SMMt} &= \frac{[(1-2u_{IIt})\cdot p_{II} - p_{PI} + 1]\cdot p_{t-1}}{[(1-2u_{IIt})\cdot p_{II} - p_{PI} + 1]\cdot p_{t-1} + [(1-2v_{IIt})\cdot p_{II} + p_{PI} + 1]\cdot(1-p_{t-1})}.
\end{aligned}$$

Using the results of Section 3.2, equations (8), regarding the probabilities u_{IIt} and v_{IIt}, the market maker's assessments P_{BMMt} and P_{SMMt} reduce to:

$$\begin{aligned}
P_{BMMt} &= \frac{[q_{HIIt}\cdot p_{II} + p_{PI} + 1]\cdot p_{t-1}}{[q_{HIIt}\cdot p_{II} + p_{PI} + 1]\cdot p_{t-1} + [(q_{LIIt}-1)\cdot p_{II} - p_{PI} + 1]\cdot(1-p_{t-1})}\\
P_{SMMt} &= \frac{[-q_{HIIt}\cdot p_{II} - p_{PI} + 1]\cdot p_{t-1}}{[-q_{HIIt}\cdot p_{II} - p_{PI} + 1]\cdot p_{t-1} + [(1-q_{LIIt})\cdot p_{II} + p_{PI} + 1]\cdot(1-p_{t-1})}.
\end{aligned}$$

These expressions can be reformulated as follows:

$$\begin{aligned}
\mathbf{P_{BMMt}} &= \frac{\mathbf{p_{t-1}\cdot(q_{HIIt}\cdot p_{II} + p_{PI} + 1)}}{\mathbf{[q_{HIIt}\cdot p_{t-1} - (1-q_{LIIt})\cdot(1-p_{t-1})]\cdot p_{II} - (1-2p_{t-1})\cdot p_{PI} + 1}}\\
\mathbf{P_{SMMt}} &= \frac{\mathbf{p_{t-1}\cdot(-q_{HIIt}\cdot p_{II} - p_{PI} + 1)}}{\mathbf{[-q_{HIIt}\cdot p_{t-1} + (1-q_{LIIt})\cdot(1-p_{t-1})]\cdot p_{II} + (1-2p_{t-1})\cdot p_{PI} + 1}}.
\end{aligned} \tag{16}$$

Hence, a complex non-linear relation arises between the assessments of the market maker and the model variables (q_{HIIt}, q_{LIIt}, p_{II}, p_{PI} and p_{t-1}).[53]

3.4 The expected profits

The periodically expected profits of each group of investors can be ascertained on the basis of the expected risky value. The latter is expressed as:

$$E[V_t|x_{gt}, h_{t-1}] = V_H \cdot P(V_t = V_H|x_{gt}, h_{t-1}) + V_L \cdot P(V_t = V_L|x_{gt}, h_{t-1}),$$

with $x_{gt} \in \{B, S\}$ and $g \in \{II, PI, N, MM\}$.
We can simplify the calculus by using the values V_H and V_L from Assumption (2).

The investors are able to estimate their profits only after the market maker discloses the prices for the period.[54] In any case, they have to decide upon their actions and to place their orders before the price is fixed (which, in fact, ensues only when orders are received by the market maker). Therefore, the investors' profit estimation does not influence the price formation. If an investor buys, she pays the ask price and gets the asset; if she sells, the bid price is obtained and the asset disposed of. The market maker performs exactly the opposite actions.

- The *perfectly informed investors* form accurate expectations about the value of the risky asset:

$$\begin{aligned}
E[V_t|x_{PIt} = B, h_{t-1}] &= 1\cdot 1 + 0\cdot 0 = 1\\
E[V_t|x_{PIt} = S, h_{t-1}] &= 1\cdot 0 + 0\cdot 1 = 0.
\end{aligned}$$

In so doing, they earn the highest profits:

$$\mathbf{G_{PIt}} = \begin{cases} E[V_t|x_{PIt} = B, h_{t-1}] - X_{Bt} = \mathbf{1 - X_{Bt}} & \text{, if } x_{PIt} = B\\ X_{St} - E[V_t|x_{PIt} = S, h_{t-1}] = \mathbf{X_{St}} & \text{, if } x_{PIt} = S. \end{cases} \tag{17}$$

[53] According to Harris (2003) (p. 288), in the real market settings the market maker has indeed to ascertain the degree and the importance of the traders' information.
[54] See Section 4.1 for the corresponding formulas.

- The *users of practical decision rules* calculate the expected value of the risky asset by dint of the Bayes rule:

$$
\begin{aligned}
P(V_t = V_H | x_{IIt} = B, h_{t-1}) &= \frac{P(x_{IIt} = B | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(x_{IIt} = B | h_{t-1})} \\
&= \frac{u_{IIt} \cdot p_{t-1}}{u_{IIt} \cdot p_{t-1} + v_{IIt} \cdot (1 - p_{t-1})} \; = \; 1 - P(V_t = V_L | x_{IIt} = B, h_{t-1}) \\
P(V_t = V_H | x_{IIt} = S, h_{t-1}) &= \frac{P(x_{IIt} = S | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(x_{IIt} = S | h_{t-1})} \\
&= \frac{(1 - u_{IIt}) \cdot p_{t-1}}{(1 - u_{IIt}) \cdot p_{t-1} + (1 - v_{IIt}) \cdot (1 - p_{t-1})} \; = \; 1 - P(V_t = V_L | x_{IIt} = S, h_{t-1}).
\end{aligned}
$$

It results:

$$
\begin{aligned}
E[V_t | x_{IIt} = B, h_{t-1}] &= \frac{u_{IIt} \cdot p_{t-1}}{u_{IIt} \cdot p_{t-1} + v_{IIt} \cdot (1 - p_{t-1})} \\
E[V_t | x_{IIt} = S, h_{t-1}] &= \frac{(1 - u_{IIt}) \cdot p_{t-1}}{(1 - u_{IIt}) \cdot p_{t-1} + (1 - v_{IIt}) \cdot (1 - p_{t-1})}.
\end{aligned}
$$

They expect the following profits for a buy and a sell, respectively:

$$
\begin{aligned}
G_{IIt} &= \begin{cases} E[V_t | x_{IIt} = B, h_{t-1}] - X_{Bt} & \text{, if} \quad x_{IIt} = B \\ X_{St} - E[V_t | x_{IIt} = S, h_{t-1}] & \text{, if} \quad x_{IIt} = S \end{cases} \\
&= \begin{cases} \dfrac{u_{IIt} \cdot p_{t-1}}{u_{IIt} \cdot p_{t-1} + v_{IIt} \cdot (1 - p_{t-1})} - X_{Bt} & \text{, if} \quad x_{IIt} = B \\ X_{St} - \dfrac{(1 - u_{IIt}) \cdot p_{t-1}}{(1 - u_{IIt}) \cdot p_{t-1} + (1 - v_{IIt}) \cdot (1 - p_{t-1})} & \text{, if} \quad x_{IIt} = S. \end{cases}
\end{aligned}
$$

Considering the simplifying formulas (8) derived in Section 3.2, we express the profits of the imperfectly informed traders in the following form:

$$
\mathbf{G_{IIt}} = \begin{cases} \dfrac{(\mathbf{1} + \mathbf{q_{HIIt}}) \cdot \mathbf{p_{t-1}}}{(\mathbf{1} + \mathbf{q_{HIIt}}) \cdot \mathbf{p_{t-1}} + \mathbf{q_{LIIt}} \cdot (\mathbf{1} - \mathbf{p_{t-1}})} - \mathbf{X_{Bt}} & \text{, if} \quad x_{IIt} = B \\ \mathbf{X_{St}} - \dfrac{(\mathbf{1} - \mathbf{q_{HIIt}}) \cdot \mathbf{p_{t-1}}}{(\mathbf{1} - \mathbf{q_{HIIt}}) \cdot \mathbf{p_{t-1}} + (\mathbf{2} - \mathbf{q_{LIIt}}) \cdot (\mathbf{1} - \mathbf{p_{t-1}})} & \text{, if} \quad x_{IIt} = S. \end{cases} \tag{18}
$$

- Having no new information at their disposal, *noise traders* behave in a manner similar to members of general public, being only aware of past market evolution:

$$
\begin{aligned}
P(V_t = V_H | x_{Nt} = B, h_{t-1}) &= \frac{P(x_{Nt} = B | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(x_{Nt} = B | h_{t-1})} \\
&= \frac{1/2 \cdot p_{t-1}}{1/2} \; = \; p_{t-1} \; = \; 1 - P(V_t = V_L | x_{Nt} = B, h_{t-1}) \\
P(V_t = V_H | x_{Nt} = S, h_{t-1}) &= \frac{P(x_{Nt} = S | V_t = V_H, h_{t-1}) \cdot P(V_t = V_H | h_{t-1})}{P(x_{Nt} = S | h_{t-1})} \\
&= \frac{1/2 \cdot p_{t-1}}{1/2} \; = \; p_{t-1} \; = \; 1 - P(V_t = V_L | x_{Nt} = S, h_{t-1}).
\end{aligned}
$$

Consequently, we obtain:

$$
\begin{aligned}
E[V_t | x_{Nt} = B, h_{t-1}] &= p_{t-1} \\
E[V_t | x_{Nt} = S, h_{t-1}] &= p_{t-1}.
\end{aligned}
$$

The action undertaken has no influence on the expectations of the noise traders.

Their profits simplify to:

$$\mathbf{G_{Nt}} = \begin{cases} E[V_t|x_{Nt}=B,h_{t-1}] - X_{Bt} = \mathbf{p_{t-1}} - \mathbf{X_{Bt}} & \text{, if } \quad x_t = B \\ X_{St} - E[V_t|x_{Nt}=S,h_{t-1}] = \mathbf{X_{St}} - \mathbf{p_{t-1}} & \text{, if } \quad x_t = S. \end{cases} \tag{19}$$

- The profit estimation in the case of the *market maker* results analogously:

$$\mathbf{G_{MMt}} = \begin{cases} -\mathbf{P_{BMMt}} + \mathbf{X_{Bt}} & \text{, if } \quad x_{gt} = B \\ -\mathbf{X_{St}} + \mathbf{P_{SMMt}} & \text{, if } \quad x_{gt} = S, \end{cases} \tag{20}$$

with $g = \{II, PI, N\}$.
This expression can be reformulated subject to the variables of interest q_{HIIt}, q_{LIIt} and p_{II}, by means of the formulas (16) for the assessments P_{BMMt} and P_{SMMt} derived in Section 3.3.[55]

4 Results

The assumptions and the intermediary results derived in the previous section allow us to compute the price formulas and subsequently analyze the influence of users of practical decision rules on prices.

4.1 Price formation

According to Assumption (8), the market maker fixes the periodical ask and bid price competitively, i.e. making no profits from accommodating the buy and sell orders received. The prices should thus equal the expected values of the risky asset[56], conditional on the market maker's current information (consisting of the common information and the contemporaneous action[57]).

- The **ask price**
  The buy price relies on the assessments of the market maker with regard to a good economic situation, assuming that the investors want to buy.

$$G_{BMMt} = 0 \quad \Rightarrow \quad \mathbf{X_{Bt}} = E[V_t|x_t = B, h_{t-1}] = \mathbf{P_{BMMt}},$$

  More precisely, the ask satisfies:

$$\mathbf{X_{Bt}} = \frac{\mathbf{q_{HIIt} \cdot p_{t-1} \cdot p_{II} + p_{t-1} \cdot p_{PI} + p_{t-1}}}{\mathbf{[q_{HIIt} \cdot p_{t-1} - (1 - q_{LIIt}) \cdot (1 - p_{t-1})] \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI} + 1}}. \tag{21}$$

- The **bid price**
  The sell price reflects the assessments of the market maker with regard to a bad economic situation, if the investors want to sell.

$$G_{SMMt} = 0 \quad \Rightarrow \quad \mathbf{X_{St}} = E[V_t|x_t = S, h_{t-1}] = \mathbf{P_{SMMt}}.$$

[55] See the price formulas from Section 4.1.
[56] Glosten and Milgrom (1985) (p. 79) and Easley and O'Hara (1987) (p. 76) make similar inferences, while Kyle (1985) (p. 1316) and Easley, Kiefer, O'Hara and Paperman (1996) (p. 1408) use exactly the same calculation method.
[57] The contemporaneous action consists of a buy for the determination of the ask prices, and of a sell in the case of the bid price, respectively.

More precisely, the bid satisfies:

$$\mathbf{X_{St}} \quad = \quad \frac{-\mathbf{q_{HIIt}} \cdot \mathbf{p_{t-1}} \cdot \mathbf{p_{II}} - \mathbf{p_{t-1}} \cdot \mathbf{p_{PI}} + \mathbf{p_{t-1}}}{[-\mathbf{q_{HIIt}} \cdot \mathbf{p_{t-1}} + (\mathbf{1} - \mathbf{q_{LIIt}}) \cdot (\mathbf{1} - \mathbf{p_{t-1}})] \cdot \mathbf{p_{II}} + (\mathbf{1} - \mathbf{2p_{t-1}}) \cdot \mathbf{p_{PI}} + \mathbf{1}}. \tag{22}$$

The ask and bid prices thus represent non-linear functions of the informational precision (revealed by q_{HIIt} and q_{HIIt}) and of the proportion of users of practical decision rules (expressed as p_{II}).

The price formation process is characterized by a double price setting in every trade period (i.e. the market maker simultaneously sets one buy price and another different sell price). The difference between the two prices represents the **bid-ask spread**, expressed as:

$$\begin{aligned} \mathbf{S_t} &= \mathbf{X_{Bt}} - \mathbf{X_{St}} \\ &= \frac{\mathbf{2p_{t-1}} \cdot (\mathbf{1} - \mathbf{p_{t-1}}) \cdot [\mathbf{p_{II}} \cdot (\mathbf{1} + \mathbf{q_{HIIt}} - \mathbf{q_{LIIt}}) + \mathbf{2p_{PI}}]}{\mathbf{1} - [\mathbf{q_{HIIt}} \cdot \mathbf{p_{t-1}} \cdot \mathbf{p_{II}} - (\mathbf{1} - \mathbf{q_{LIIt}}) \cdot (\mathbf{1} - \mathbf{p_{t-1}}) \cdot \mathbf{p_{II}} - (\mathbf{1} - \mathbf{2p_{t-1}}) \cdot \mathbf{p_{PI}}]^{\mathbf{2}}}. \end{aligned} \tag{23}$$

With regard to the bid-ask spread we formulate the following statement:

Proposition 1.: *The bid-ask spread is always positive.*
Proof: See Appendix A.

Note: Both the ask and bid prices take values in the interval $[0, 1]$, because they result directly from the probabilities P_{BMMt} and P_{BMMt}, respectively. Considering Proposition 1., the bid-ask spread moves between a minimal value of 0 and a maximal value of 1.

The formation of a positive bid-ask spread cannot be accounted for by order processing or inventory costs (which are kept constant, according to Assumption (8)). The only remaining explanation is to be found in the adverse selection costs the market maker has to pay to an increased extent when trading with better informed agents. She tries to protect herself against losses from such trade by augmenting the ask and lowering the bid. These modified trade conditions affect all agents in the market, thus allowing the market maker to recover her losses from the trade with the informed at the expense of the uninformed traders).[58] These ideas are summarized by the following corollary:

Corollary: *The positive bid-ask spread of a given trade period is the result of the market makers' reaction to the adverse selection problem generated by the informational asymmetries present in the market.*
Proof: See Appendix A.

The bid-ask spread expansion as a consequence of the increasing adverse selection is consistent with the evidence provided by other authors.[59] Our result (as expressed in the Corollary) confirms this conclusion and speaks for the model's validity.

[58]See Kyle (1985) (p. 1316) and O'Hara (1995) (p. 54). In the original interpretation of Akerlof (1970), the adverse selection points to the following problem: when buyers cannot precisely infer the quality of the products in a market, the average quality of the entire supply deteriorates. Accordingly, if the market maker in our model cannot accurately evaluate the information degree of her counter-parties, the transaction terms worsen for all traders.

[59]See Glosten and Milgrom (1985) (pp. 89-91), Kyle (1985) (pp. 1319-1320, 1322-1323 and 1327-1328), Easley and O'Hara (1987) (pp. 80-81) or Easley, Kiefer, O'Hara and Paperman (1996) (pp. 1411-1412). Glosten and Harris (1988) (pp. 135 and 140) emphasize the importance of the adverse selection spread component in comparison to the inventory costs. While the inventory effects on the bid-ask spread are transitory, the influence of the adverse selection exhibits a permanent character (p. 131). Moreover, Van Ness, Van Ness and Warr (2001) (pp. 20-21) show that the adverse selection decreases with an increasing number of market makers trading an asset. This should draw attention to the increased danger of the informational asymmetry in markets with only one market maker per asset.

4.2 The influence of users of practical decision rules on prices

In order to examine the price variation subject to the variables of interest (i.e. q_{HIIt}, q_{LIIt} and p_{II}), we make several further remarks and assumptions:

- The probabilities q_{HIIt} and q_{LIIt} (that the users of practical trading rules receive a positive signal in a good, and in a bad economic situation, respectively) can be considered *independent*.[60]
- (11) Assumption: The *proportion of the perfectly informed traders* p_{PI} is *constant.*
- (12) Assumption: The market maker keeps the *a-priori assessment* p_{t-1} *constant in every period* (i.e. at the level p_0).[61]

As a direct consequence of these assumptions, the market makers' assessments P_{BMMt} and P_{SMMt} and therefore the transaction prices X_{Bt} and X_{St} will be governed by three variables only, namely the qualitative and quantitative characteristics of the users of practical decision rules: q_{HIIt}, q_{LIIt} and p_{II}.

Both hypothesized effects of the supporters of empirical methods are thus readily identifiable: the probabilities q_{HIIt} and q_{LIIt} refer to an influence by means of informational precision, while p_{II} point to a quantitative aspect, induced by the proportion of these traders to the totality of investors in the market. The following statement summarizes the implications of the c.p. variation of these two factors for the ask and bid prices (if the market maker regards the users of practical decision rules as informed):

Proposition 2.: *A c.p. intensification of method-based trade (either on the basis of more precise information or by means of an increased fraction of the users of practical decision rules) causes the deterioration of trading terms for all market participants (i.e. an augmentation of the ask price and a diminution of the bid price).*

A possible explanation for the emerging spread fluctuation is provided by the informational asymmetry between the investors and the market maker. According to Assumption (9), the market maker is unaware of the identity of a potential counter-party. She is acquainted only with the fact that doing business with better informed agents generates losses.[62] Consequently she faces an *adverse selection problem.*[63]

We subsequently reformulate and then demonstrate Proposition 2. by means of several further concrete statements, focusing on the effects of the c.p. variation of each variable of interest (q_{HIIt}, q_{LIIt} and p_{II}, respectively).

- When the economy is in a good state, the accuracy of the method-based information depends positively on q_{HIIt}. The effects of the c.p. variation of this probability on the prices are outlined in

[60] This assumption enables the analysis of the price movements generated by the c.p. variation of the probabilities q_{HIIt} and q_{LIIt}.

[61] In reality the market maker can assess this a-priori probability on the basis of the received buy and sell orders. If b_t and a_t represent the fractions of the buy and of the sell orders, respectively, received at t (with $a_t + b_t = 1$), the market maker can equate the buy probability (independently of the economic situation) with b_t, so that: $P(x_t = B|h_{t-1}) = u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1 - p_{t-1}) = b_t$. The unknown probability p_{t-1} satisfies: $p_{t-1} = \frac{b_t - v_{MMt}}{u_{MMt} - v_{MMt}} = \frac{2b_t - (q_{LIIt}-1)\cdot p_{II} + p_{PI} - 1}{(q_{LIIt} - q_{HIIt}+1)\cdot p_{II} + 2p_{PI}}$. Consequently, p_{t-1} depends on p_{II}, q_{HIIt} and q_{LIIt}. The resulting endogenity of p_{t-1} notably complicates the analysis and, consequently, will not be further considered.

[62] See Copeland and Galai (1983) (pp. 1461-1462), Easley and O'Hara (1987) (p. 73) or Madhavan (2000) (p. 216). In our approach the market maker can can only evaluate the proportion of informed traders in the market by dint of the variables p_{PI}, p_{II}, q_{HIIt} and q_{LIIt}.

[63] See Glosten and Milgrom (1985) (p. 82-84) and the Corollary of Proposition 1. in Section 4.1.

the following proposition:

Proposition 2.1.: *The c.p. variation of the probability q_{HIIt} (that the users of practical decision rules deduce a positive signal in a good economic situation) determines a directly proportional and concave variation of the ask price. In turn, the bid price runs inversely proportional, but is additionally concave. Consequently, the bid-ask spread exhibits a positive dependency on this c.p. variation, with a convex evolution only for sufficiently high values of q_{HIIt}.*
Proof: See Appendix A.

In other words, an increase in the probability q_{HIIt} (other things being equal) causes the probability P_{BMMt} the market maker assesses for a buy to rise, while the sell probability in a good economic situation P_{SMMt} falls. The ask price X_{Bt} augments accordingly (at first faster, then more slowly), while the bid price X_{St} diminishes (at first more slowly, then faster). Consequently, the bid-ask spread S_t rises, but its increasing speed depends on the values of the model variables.[64] The bid-ask spread rises faster for higher values of informational precision (expressed by q_{HIIt}).

The buy probability of the imperfectly informed traders during a positive economic development increases c.p. with the probability of a positive signal. Consequently, the market maker counts on an augmentation of the probability of making losses and safeguards by enhancing the bid-ask spread. The rise speed of the spread increases c.p. more for a higher probability q_{HIIt}, because the market maker reacts more strongly as a result of her counter-parties being better informed.

Figures 1 and 2 in Appendix B illustrate the evolution of the ask and bid prices, as well as of the spread, in the particular cases with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{LIIt} = 0.4$ and $p_{II} = 0.1$, respectively $p_{II} = 0.8$.[65] As stated in Proposition 2.1., the ask increases and the bid diminishes in both situations considered. The corresponding curvatures are readily observable for a higher fraction of the users of practical decision rules. The bid-ask spread runs convexly in the first case ($p_{II} = 0.1$). Even for quite small values of q_{HIIt}, the curvature parameter α[66] is not great enough to change this evolution. In the second case ($p_{II} = 0.8$), the bid-ask spread reaches an inflection point.[67] Moreover, the bid-ask spread is higher in the (second) case with more users of practical decision rules. According to the Corollary of Proposition 1., this can rely on increased adverse selection costs.

- The precision of the method-based information during a bad economic period is negatively related to the probability q_{LIIt}. The c.p. effect of this probability on the prices can be outlined as follows:

Proposition 2.2.: *A c.p. increase in the probability q_{LIIt} (that the users of practical decision rules receive a positive signal, conditional upon a low value of the risky asset) causes a convex diminishment of the ask price and a convex augmentation of the bid price, respectively. Consequently, the bid-ask spread falls. Its evolution is more probably convex for lower values of q_{LIIt}.*

[64] For a detailed description of the spread evolution see the proof of Proposition 2.1. in Appendix A.
[65] Mathematica 5.0 was used for generating the graphics.
[66] See the proof of Proposition 2.1. in Appendix A.
[67] If $p_{II} = 0.1$, then $\alpha = 0.06 \cdot q_{HIIt} - 0.014$. Consequently, we have $\alpha < 0$, for $q_{HIIt} < 0.233$. Even for the minimal allowable value $q_{HIIt} = 0$, α satisfies $\alpha \cdot (3+\alpha^2) + [p_{PI} + p_{II} \cdot (1 - q_{LIIt})] \cdot (1+3\alpha^2) = 0.068 \geq 0$. The spread is thus convex for all values of $q_{HIIt} = 0$. In the second situation ($p_{II} = 0.8$), $\alpha = 0.48 \cdot q_{HIIt} - 0.182$ and hence $\alpha < 0$ for $q_{HIIt} < 0.379$. Even for a small value $q_{HIIt} = 0.1$, we have $\alpha \cdot (3 + \alpha^2) + [p_{PI} + p_{II} \cdot (1 - q_{LIIt})] \cdot (1 + 3\alpha^2) = -0.288 \geq 0$, revealing a concave spread evolution. The bid-ask spread reaches thus an inflexion point for small values of the probability q_{HIIt}.

Proof: See Appendix A.

The increase of the probability q_{LIIt} (other things being equal) causes a contrary movement of prices. The buy probability P_{BMMt} and, therefore, the ask X_{Bt} fall (at first faster, then more slowly), while the sell probability P_{SMMt} and the bid X_{St} rise (at first more slowly, then faster). The bid-ask spread S_t decreases accordingly. Its variation speed depends on the values of the model variables, the spread thus evolving convexly or reaching an inflexion point.[68] The more precise the information of the users of practical decision rules in a bad economic situation is (i.e. the lower the probability q_{LIIt} is), the faster the bid-ask spread varies.

An explanation for these movements is to be found in the fact that a positive signal during a negative state of the economy at large contains practically no useful information, therefore causing the users of practical decision rules to trade randomly. The market maker is thus confronted by less well informed traders, this being all the more probable when their information tends to be imprecise, causing the market maker to improve the trade terms for all agents as a consequence .

Figures 3 and 4 in Appendix B illustrate the evolution of the ask and bid prices, as well as of the spread, in the particular cases with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{HIIt} = 0.6$ and $p_{II} = 0.1$ and $p_{II} = 0.8$, respectively. As was demonstrated in Proposition 2.2., the ask diminishes, the bid increases and the bid-ask spread therefore decreases with the probability q_{LIIt}. The curvatures are distinct for a higher proportion of the imperfectly informed traders. For a small informational precision (i.e. a high q_{LIIt}), the curvature parameter α[69] in both cases reaches values high enough to turn the convex course into a concave one.[70] Besides, the bid-ask spread takes higher values in the (second) case with more users of practical decision rules. This fact confirms the increase of the adverse selection costs for the market maker, as stated in the Corollary to Proposition 1.

- With regard to the influence of the fraction of imperfectly informed traders on the prices, we can formulate the following statement:

Proposition 2.3.: *The ask price depends positively and convexly on the c.p. variation of the proportion of the users of practical decision rules p_{II}, while the bid price exhibits a negative and concave evolution. As a consequence, the bid-ask spread changes positively and convexly.*
Proof: See Appendix A.

In other words, the increase in the proportion of users of practical decision rules p_{II} (other things being equal) determines the buy probability P_{BMMt} to rise, and the sell probability P_{SMMt} to fall. This fact causes the increase in the ask price X_{Bt} (at first more slowly, then faster) and the decline of the bid price X_{St} (also more slowly initially, subsequently faster). Consequently, the bid-ask spread S_t rises with an increasing speed.

The evolutions of the bid-ask spread and the prices found are to be expected as a result of increased

[68] For a detailed description of the spread evolution see the proof of Proposition 2.2. in Appendix A.

[69] See the proof of Proposition 2.2. in Appendix A.

[70] For $p_{II} = 0.1$ is $\alpha = 0.04 \cdot q_{LIIt} + 0.006$. Consequently we have $\alpha > 0$, for all q_{LIIt}. For a high value of $q_{LIIt} = 0.9$ α satisfies $-\alpha \cdot (3 + \alpha^2) + [p_{PI} + p_{II} \cdot q_{HIIt}] \cdot (1 + 3\alpha^2) = -0.015 \leq 0$. The bid-ask spread has then already reached the inflexion point. In the second case ($p_{II} = 0.8$), $\alpha = 0.32 \cdot q_{LIIt} - 0.022$ and accordingly $\alpha > 0$, for $q_{LIIt} > 0.069$. For the same high value of $q_{LIIt} = 0.9$, it results $-\alpha \cdot (3 + \alpha^2) + [p_{PI} + p_{II} \cdot q_{HIIt}] \cdot (1 + 3\alpha^2) = -0.683 \leq 0$. The bid-ask spread also reaches the inflexion point in this case.

adverse selection costs, due to the augmentation of the fraction of informed traders.[71] The more users of practical decision rules trade in the market, the greater the danger of adverse selection for the market maker is, so that every further rise in the proportion of this trader category substantially accelerates the spread increase.

Figures 5 and 6 in Appendix B illustrate the evolution of the ask and bid prices, as well as of the spread, in the particular cases with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{LIIt} = 0.4$ and $q_{HIIt} = 0.6$, respectively $q_{HIIt} = 0.8$. According to Proposition 2.3., the ask increases, the bid decreases and the bid-ask spread rises with the fraction p_{II}. The curvatures are more observable for a higher informational precision in a positive economic situation q_{HIIt}.

Besides the univariate analysis we have performed so far, we have also investigated the bivariate particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$ and $q_{HIIt} = 0.6$, $q_{LIIt} = 0.4$ and/or $p_{II} = 0.1$. The results confirm the hypothesis set forth in Proposition 2. (i.e. the more intensely the users of practical decision rules trade, the worse the transaction terms for all market participants are). The increase in the ask and the decrease in the bid are more extreme for the borderline cases with: $q_{HIIt} \rightarrow 1$ and $p_{II} \rightarrow 1$, respectively, $q_{LIIt} \rightarrow 0$ and $p_{II} \rightarrow 1$, and $q_{HIIt} \rightarrow 1$ and $q_{LIIt} \rightarrow 0$, respectively. (See figures 7-9 in Appendix B.)

Furthermore, we derive from the price formulas two further global statements concerning price movements and the influence of users of practical decision rules on prices.

- **Proposition 3.**: *The case with fully incorrectly informed users of practical decision rules ($q_{HIIt} = 0$, i.e. no positive signal during a good economic situation and $1 - q_{LIIt} = 0$, i.e. no zero-signal for a low value of the risky asset) is equivalent to the case with no supporters of empirical methods ($p_{II} = 0$).*
  Proof: See Appendix A.

  Due to the fact that the signals of the completely incorrectly informed users of practical decision rules can hardly reflect the real economic situation, these traders act at random, just as the noise traders do. The groups of investors trading in the market are thus reduced in both cases to the following two: perfectly informed agents and noise traders.

  The bid-ask spread remains further positive:

$$S_t = \frac{4p_{t-1} \cdot (1 - p_{t-1}) \cdot p_{PI}}{1 - p_{PI}^2 \cdot (1 - p_{t-1})},$$

  because the market maker still has to protect herself against the fully informed agents.

- Another conclusion considers both extreme situations with $p_{t-1} = 0$ and $p_{t-1} = 1$.[72]

  Proposition 4.: *If the a-priori assessment of the market maker regarding the economic situation assumes extreme values ($p_{t-1} = 0$ or $p_{t-1} = 1$), the market maker sets the same prices for buy*

[71] According to Assumption (11) (of a constant p_{PI}), the increase in the fraction of users of practical decision rules generates the augmentation of the ratio of informed to uninformed traders $\frac{p_{II}+p_{PI}}{p_N}$. Glosten and Milgrom (1985) (p. 89) confirm the conclusion that the increase of this ratio can cause an enlargement of the bid-ask spread.

[72] Because these two cases lead to trivial results, we do not take them into consideration in connection with the previously discussed influence of the users of practical decision rules on prices.

and for sell, i.e. exactly in the amount of the value of the risky asset.

In both cases, the market maker is already in $t-1$ as well as being perfectly informed about the economic situation in t. This puts her in the position of being able to undertake buys and sells at the "right" prices (equal to the true value of the risky asset V_t).
Proof: See Appendix A.

Excluding the possibility of a good economic development in the next period (i.e. $p_{t-1} = 0$) is equivalent to the market maker's belief that the risky value during a bad period for the economy at large equals zero $V_L = 0$. She will consequently set the transaction prices at zero, thus having no incentives for trading. But if the market maker is convinced of future positive economic development (i.e. $p_{t-1} = 1$), both prices reach their maximal values: $V_H = 1$.

4.3 The misperceptions of users of practical decision rules

In comparison to the the perception of the perfectly informed traders regarding the prices, the perception of the users of practical decision rules, who receive only imperfect information, can only be biased. Following De Long, Shleifer, Summers and Waldmann (1990), we denote such a perception distortion as *misperception*.[73]

Trying to appropriately define the misperception of the supporters of empirical methods regarding the asset prices, we distinguish two different aspects of possible interest: firstly, the perception of the real economic situation, if the investors receive a certain information signal, and, secondly, the perception of the appropriate action in a given state of the economy at large. We consequently express the misperception in terms of these two aspects, taking the assessments and the actions of the perfectly informed traders as a benchmark, following Assumption (8).

Our aim is to find out to what extent the same input (i.e. the same information signals, and the same value of the risky asset, respectively) can generate different outputs (i.e. assessments and actions) of the two categories of informed traders (i.e. perfectly and imperfectly informed). The reason is to be found in a potentially stronger cumulate influence on the prices, due to more similar assessments and actions of these two investor groups.

The *misperception of the economic situation* draws upon the differences in the assessments of the imperfectly and perfectly informed investors when they receive the same information signal. We define it differently with regard to a good, and a bad state of the economy at large, respectively:

$$\begin{aligned}
\rho_{IIt}^{V_H} &= P(V_t = V_H | s_{PIt} = 1, h_{t-1}) - P(V_t = V_H | s_{IIt} = 1, h_{t-1}) \\
&= P_{1PIt} - P_{1IIt} = 1 - P_{1IIt} \\
\rho_{IIt}^{V_L} &= P(V_t = V_L | s_{PIt} = 0, h_{t-1}) - P(V_t = V_L | s_{IIt} = 0, h_{t-1}) \\
&= (1 - P_{0PIt}) - (1 - P_{0IIt}) = 1 - (1 - P_{0IIt}) = P_{0IIt}.
\end{aligned}$$

[73] The approach of De Long, Shleifer, Summers and Waldmann (1990) (p. 708) discusses the misperception of the noise traders (a category which includes the technical analysts as well, see p. 706) regarding the expected prices of the risky asset. This misperception is modelled as a random variable. We define the misperception in connection with two central elements of our model: the assessments of the not fully informed traders with regard to the economic situation, and their actions in a given state of the economy, respectively.

Falling back on the results (13) of Section 3.3, we obtain:

$$
\begin{aligned}
\rho_{\mathbf{IIt}}^{\mathbf{V_H}} &= \mathbf{1} - \frac{\mathbf{q_{HIIt}} \cdot \mathbf{p_{t-1}}}{\mathbf{q_{HIIt}} \cdot \mathbf{p_{t-1}} + \mathbf{q_{LIIt}} \cdot (\mathbf{1} - \mathbf{p_{t-1}})} \\
\rho_{\mathbf{IIt}}^{\mathbf{V_L}} &= \frac{(\mathbf{1} - \mathbf{q_{HIIt}}) \cdot \mathbf{p_{t-1}}}{(\mathbf{1} - \mathbf{q_{HIIt}}) \cdot \mathbf{p_{t-1}} + (\mathbf{1} - \mathbf{q_{LIIt}}) \cdot (\mathbf{1} - \mathbf{p_{t-1}})}.
\end{aligned}
\tag{24}
$$

The *misperception of the appropriate action* is defined as the difference between the actions of the imperfectly and perfectly informed investors in a given economic situation:

$$
\begin{aligned}
\rho_{IIt}^{B} &= P(x_{PIt} = B|V_t = V_H, h_{t-1}) - P(x_{IIt} = B|V_t = V_H, h_{t-1}) \; = \; 1 - u_{IIt} \\
\rho_{IIt}^{S} &= P(x_{PIt} = S|V_t = V_L, h_{t-1}) - P(x_{IIt} = S|V_t = V_L, h_{t-1}) \; = \; 1 - (1 - v_{IIt}) \; = \; v_{IIt}.
\end{aligned}
$$

The results (8) from Section 3.2 help in obtaining the following linear expressions:

$$
\begin{aligned}
\rho_{\mathbf{IIt}}^{\mathbf{B}} &= \frac{\mathbf{1} - \mathbf{q_{HIIt}}}{\mathbf{2}} \\
\rho_{\mathbf{IIt}}^{\mathbf{S}} &= \frac{\mathbf{q_{LIIt}}}{\mathbf{2}}.
\end{aligned}
\tag{25}
$$

According to the previous calculation and to Assumptions (11) and (12), the two misperception forms can be represented as functions of the two probabilities of interest (describing the informational accuracy of the method-based signals): q_{HIIt} and q_{LIIt}. The buy misperception ρ_{IIt}^{B} depends solely on the probability of a positive signal. This is due to the fact that, in a good economic situation, only such signals can generate the right action (namely buy) of the users of practical decision rules. Analogously, the sell misperception ρ_{IIt}^{S} depends only on the probability of a zero-signal in a bad state of the economy q_{LIIt}.

The c.p. variation of the probabilities q_{HIIt} and q_{LIIt}, respectively, causes different movements of the misperceptions, as summarized in the following proposition:

Proposition 5.: *The increased informational accuracy of the method-based signals (c.p. expressed through the probabilities q_{HIIt} and q_{LIIt} in a good and a bad economic situation, respectively) determines an improvement of the perception of the users of practical decision rules in comparison to the perception of the perfectly informed investors, both concerning their assessments and the appropriate action. The single forms of misperception exhibit specific evolutions.*

Two statements focussing on the c.p. variations of the probabilities q_{HIIt} and q_{LIIt}, respectively, facilitate the more concrete formulation and the subsequent demonstration of Proposition 5..

- The following proposition outlines the effects of the c.p. variation of q_{HIIt} on the misperceptions.

  Proposition 5.1.: *The c.p. variation of the probability q_{HIIt} (that the users of practical decision rules obtain a positive information signal conditional upon a high value of the risky asset) causes the decrease of the misperceptions regarding the economic situation. The evolution is convex for the misperception concerning the good state of the economy $\rho_{IIt}^{V_H}$ and concave for the one regarding the bad economic development. The misperception of the appropriate action in a good economic situation ρ_{IIt}^{B} falls linearly with an increasing q_{HIIt}, while the sell misperception ρ_{IIt}^{S} remains independent of this probability.*
  Proof: See Appendix A.

In other words, a c.p. augmentation of the probability q_{HIIt} generates an increase of the assessment P_{1IIt} regarding the high value of the risky asset, and, conversely, a decrease of the assessment P_{0IIt} concerning a low risky value. Accordingly, both misperceptions of the economic situation fall, namely $\rho^{V_H}_{IIt}$ initially faster, subsequently more slowly, and $\rho^{V_L}_{IIt}$ reciprocally, i.e. initially more slowly, subsequently faster. We expect this result on account of an increased chance of the users of practical decision rules delivering correct assessments when receiving more accurate information. The decline of the buy misperception ρ^{B}_{IIt} as a result of an increasing q_{HIIt} is due to the fact that a more precise method-based piece of information results more probably in an appropriate action.

- The c.p. variation of q_{LIIt} causes related misperception movements, which are summarized in the following proposition:

  Proposition 5.2.: *The misperception with regard to a good economic situation $\rho^{V_H}_{IIt}$ depends c.p. positively and concavely on the probability q_{LIIt} (that the users of practical decision rules receive a positive signal if the value of the risky asset is low), while the misperception concerning a poor state of the economy at large $\rho^{V_L}_{IIt}$ also exhibits a c.p. positive, but convexly relation to the probability q_{LIIt}. The sell misperception ρ^{S}_{IIt} increases linearly with the c.p. augmentation of q_{LIIt}, while the buy misperception ρ^{B}_{IIt} does not change.*
  Proof: See Appendix A.

  Because an increase of q_{LIIt} points to a less accurate method-based information, the assessments and actions relying on such information are distorted all the more strongly. The variation speed of $\rho^{V_H}_{IIt}$ is initially higher and subsequently lower, and develops exactly in the opposite manner for $\rho^{V_L}_{IIt}$ remaining constant with regard to ρ^{S}_{IIt}.

Although we obviously expect a decline in misperception due to an improved accuracy of the method-based information (as outlined in Proposition 5.), we should note the different evolutions of the single misperception forms. Because the actions of the investors are directly based on their information, the misperceptions of the right action ρ^{B}_{IIt} and ρ^{S}_{IIt} depend linearly on the accuracy of the method-based information (expressed through q_{HIIt} and q_{LIIt}). In respect of the assessments, a joint effect of the contemporaneous and previous information arises (see Section 3.3), so that the misperceptions $\rho^{V_H}_{IIt}$ and $\rho^{V_L}_{IIt}$ exhibit complex evolutions due to the c.p. variation of q_{HIIt} and q_{LIIt}, respectively. Although q_{HIIt} (q_{LIIt}) refers only to a positive (negative) economic situation, the two probabilities influence both the misperception regarding a good state of the economy $\rho^{V_H}_{IIt}$, and on the misperception concerning a low value of the risky asset $\rho^{V_L}_{IIt}$. For a low informational accuracy (i.e. low values of q_{HIIt}, and high values of q_{LIIt}, respectively), we stress a more sensitive variation of $\rho^{V_H}_{IIt}$. The variation speed of $\rho^{V_L}_{IIt}$ exhibits an opposite evolution.

Both misperception forms indicate a distortion of the price perception by the users of practical decision rules. This can be ascribed to the fact that both a positive signal in a good phase and the buy generated by such a signal express an expected price increase in the coming period. Analogously, the method-based signals with a zero-value, as well as the sells can be associated with an expected price fall. In keeping with De Long, Shleifer, Summers and Waldmann (1990) (p. 711) the misperception of the expected prices generates real price distortions.[74] Our results provide a starting point for a further and more detailed analysis of the direct relation between prices and misperceptions.

[74] De Long, Shleifer, Summers and Waldmann (1990) (p. 705f.) assume the existence of two groups of agents (the

5 Conclusions

The **results** of our approach can be summarized as follows:

- The informational asymmetry in a market with one market maker, perfectly informed investors, imperfectly informed users of practical decision rules and uninformed noise traders affects the market prices by generating a positive bid-ask spread. (Proposition 1. and the Corollary demonstrate this result.)
- The c.p. intensification of the method-based trade (either on the basis of a higher informational accuracy, or by means of an increased proportion of the users of practical decision rules) determines the intensification of the adverse selection and hence a degradation of the general transaction terms. (This conclusion is demonstrated in Proposition 2..)
- The biased price perception of the users of practical decision rules in comparison to the perfectly informed traders is expressed twofold: with respect to the economic situation given a certain information signal, and regarding the appropriate action in a given state of the economy at large. As expected, the distortion (called misperceptions) is lower for a higher accuracy of the method-based information. The single misperception-forms, however, exhibit different curvatures (i.e. non-linear for the misperception of the economic situation and linear in the case of the appropriate action). (This results from Proposition 5.)

The model provides further insight into the microstructure of the financial markets, explicitly considering the influence of the users of practical decision rules on the prices. In order to quantify this influence by modelling the price formation process, we make several simplifying assumptions, which, on the one hand, serve to clarify our conclusions, but, on the other hand, fail to give full account of the complexity of the processes described in comparison to the real phenomena.

The **model** can be **extended** to incorporate *variable order sizes.*[75] The hypothesized existence of perfectly informed traders can be ruled out by allowing for an arbitrarily high order size (and maintaining the other assumptions).[76] Moreover, if the order size is variable, the model can account for the influence of the trade volume.[77] The unknown probabilities q_{HIIt} and q_{LIIt} could then be formulated conditional upon the volume and thus rendered endogenous. Furthermore, the variable order size increases the risk and the costs of the market makers's inventory, hence affecting the prices.[78]

uninformed, short-term oriented noise traders and the informed, long-term oriented sophisticated investors), who trade a risky asset, and model the misperception as a normally distributed random variable. They show that the misperception of the noise traders regarding the expected prices generates a new risk form (the so called noise trader risk). The noise traders unconsciously take this risk and are consequently rewarded in excess, i.e. the greater their misperception is. Easley and O'Hara (2001) (p. 17) reach the same conclusion. They further demonstrate that private information determines the appearance of a new risk type and therefore causes an increase of the expected returns by the noise traders, who take this risk, because they cannot ascertain comprehensive information about the risky asset from the market price.

[75] Glosten and Harris (1988) (p. 135) outline the increased influence of adverse selection on the prices with the order size. Easley and O'Hara (1987) (pp. 74-76) demonstrate the existence of two possible market equilibria, if the informed investors have the choice between a large and a small order size: i.e. the one with informed investors transacting only small orders, the other with informed investors placing both small and large orders. The most important surveys with regard to the influence of the order size on prices and on the relation between the volume and the volatility are summarized in Coughenour and Shastri (1999) (pp. 8-10).

[76] In this case, the information will be quickly (right from the first trade period, in fact) incorporated into the prices.

[77] Blume, Easley and O'Hara (1994) emphasize the role of the transaction volume for the learning process and thereby for the price adjustment to the new information. Wang (1994) ascertains an indirectly proportional relation between the volume and the informational asymmetry, taking into consideration the fact that the transaction volume can be important especially in the case of technical analysts as users of practical decision rules, because their method is based upon the combination of prices and volume.

[78] This results in the so called inventory component of the bid-ask spread. Coughenour and Shastri (1999) (pp. 2-5) provide an overview of the surveys regarding the different spread components (due to adverse selection, inventory and order

A further relevant development should consider *more complex decision rules* for the single categories of investors (e.g. profit maximization).[79] The investors could be allowed to issue both *market and limit orders*.[80]

Another subject for future research concerns the question of *market efficiency*. Some of the classical microstructure models [81] demonstrate that prices are martingales (i.e. that the information available in the market at a certain moment in time can provide no information about future prices). An analogous calculation using our approach provides the following result: *The market we analyze can be weak-form efficient only under the condition that the market maker formulates the same a-priori assessment p_{t-1} in every period.* (See Appendix A for the proof.) Although this condition can be theoretically fulfilled in the framework of the model, it will probably not be met in a real market setting.[82] Furthermore, an appropriate judgement of market efficiency necessitates a dynamic analysis of the price formation and represents an interesting subject for future research.[83]

processing). The inventory models focus on this inventory spread component, as can be inferred from the summaries of O'Hara (1995) (pp. 13-52), Madhavan (2000) (pp. 213-215) or Biais, Glosten and Spatt (2002) (pp. 6-7 and 15-16). The models with strategic investors analyze the problem of variable order size in a market with adverse selection. See O'Hara (1995), (p. 89-151) and Biais, Glosten and Spatt (2002) (p. 8-11, 16-18 and 22-26).

[79] We can, moreover, assume that the informed investors strategically choose the trading moment. See Admati and Pfleiderer (1988).

[80] The market orders provide an execution at the market price, while the limit orders can be executed only up to a given price limit (for a buy) or from a given price limit (for a sell). Because the limit orders remain active for several periods (thus being anytime executable against a fresh market order at any given time), they compete with the prices fixed by the market maker. See Madhavan (2000) (p. 229). According to O'Hara (1995) (p. 46-47) the execution probability of a limit order affects the bid-ask spread.

[81] See Glosten and Milgrom (1985) (p. 82-83) and Easley and O'Hara (1987) (p. 86-87).

[82] The market maker probably estimates p_{t-1} on the basis of the received orders. See Section 4.2, footnote 61.

[83] Fundamental decision rules, which draw upon both past and contemporaneous public information, retain their usefulness in a weak-form efficient market. Within the framework of our model, even technical analysis could be considered a rational support in decision making. There are several reasons in favor of this. First, common information consists only of past prices and actions of the respective trading interval, but not of the information of previous intervals. Nevertheless, technical analysts take into account such previous information (it influences their decisions through the probabilities q_{HIIt} and q_{LIIt}). Concerning the real market settings, we should underline that the public availability of a particular piece of information is not necessarily equivalent to its actual use in the process of decision making. Second, the particular combination of different past data (such as prices and volumes) can create additional value for the different technical indicators. (Treynor and Ferguson (1985) (p. 768) plead for the usefulness of past prices in a weak-form efficient setting for computing the probability, that one investor receives private information before the market. Brown and Jennings (1990) (pp. 536-538) demonstrate that the technical analysis extracts its value from using a price sequence in place of single prices. The efficiency of the market depends within this framework on the definition adopted (pp. 541-542). Blume, Easley and O'Hara (1994) (pp. 171-172) stress the informative character of a sequence of prices and volumes, which makes technical analysis a natural component of the learn process. In the approach of Kavajecz and Odders-White (2002) the technical analysis proves to be a practical method for estimating the liquidity supplied on the limit order book.)

A Appendix - Proofs

Proposition 1. - Proof:

$$\begin{aligned}
X_{Bt} - X_{St} &= (P_{BMMt} - P_{SMMt}) \\
&= \frac{u_{MMt} \cdot p_{t-1}}{u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1-p_{t-1})} - \frac{(1-u_{MMt}) \cdot p_{t-1}}{(1-u_{MMt}) \cdot p_{t-1} + (1-v_{MMt}) \cdot (1-p_{t-1})} \\
&= \frac{(u_{MMt} - v_{MMt}) \cdot (1-p_{t-1})}{[u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1-p_{t-1})] \cdot [(1-u_{MMt}) \cdot p_{t-1} + (1-v_{MMt}) \cdot (1-p_{t-1})]} \\
&= \frac{[(q_{HIIt} - q_{LIIt} + 1) \cdot p_{II} + 2p_{PI}] \cdot (1-p_{t-1})}{2[u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1-p_{t-1})] \cdot [(1-u_{MMt}) \cdot p_{t-1} + (1-v_{MMt}) \cdot (1-p_{t-1})]}.
\end{aligned}$$

Because both q_{HIIt}, q_{LIIt}, p_{II} and p_{PI}, and also u_{MMt}, v_{MMt} and p_{t-1} are probabilities (i.e. they take values in the interval $[0,1]$), there results:

$$\mathbf{X_{Bt}} \geq \mathbf{X_{St}}.$$

Corollary - Proof:

If no informed investor trades in the market, i.e. $p_N = 1$ (or $p_{PI} + p_{II} = 0$), the positive fractions $p_{PI} = p_{II} = 0$. In this case, the prices refer only to past information and are thus equal: $X_{Bt} = X_{St} = p_{t-1}$. The bid-ask spread will accordingly be $S_t = 0$.

If there are either perfectly or imperfectly informed investors trading in the market, but no uninformed traders (i.e. $p_N = 0$, that means $p_{PI} + p_{II} = 1$), the bid-ask spread is positive. As long as $p_{PI} \neq 0$, the spread does not reach its maximal value, but only:

$$S_t = \frac{[(q_{HIIt} - q_{LIIt} + 1) \cdot p_{II} + 2p_{PI}] \cdot (1-p_{t-1})}{2[u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1-p_{t-1})] \cdot [(1-u_{MMt}) \cdot p_{t-1} + (1-v_{MMt}) \cdot (1-p_{t-1})]} \geq 0.$$

If only perfectly informed investors trade in the market, i.e. either $[(p_{PI} = 1) \wedge (p_{II} = 0)]$ or $[(p_{PI} = 0) \wedge (p_{II} = 1) \wedge (q_{HIIt} = 1) \wedge (q_{LIIt} = 0)]$, the buy price is maximal $X_{Bt} = 1$ and the sell price minimal $X_{St} = 0$. The bid-ask spread reaches its maximal value (namely 1), too.[84]
We can consequently conclude that a greater informational asymmetry between the investors and the market maker causes an increased bid-ask spread.

Proposition 2.1. - Proof:

The computation of the first two partial derivatives of the buy and sell prices with respect to the probability q_{HIIt} provides the following results:

$$\frac{\partial X_{Bt}}{\partial q_{HIIt}} = \frac{\partial P_{BMMt}}{\partial q_{HIIt}} = \frac{p_{t-1} \cdot (1-p_{t-1}) \cdot p_{II} \cdot [1 - p_{PI} - p_{II} \cdot (1-q_{LIIt})]}{[q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1-q_{LIIt}) \cdot (1-p_{t-1}) \cdot p_{II} - (1-2p_{t-1}) \cdot p_{PI} + 1]^2}.$$

Because $1 - p_{PI} - p_{II} = p_N > 0$ and all variables take values in the interval $[0,1]$, we have:

$$\frac{\partial X_{Bt}}{\partial q_{HIIt}} \geq 0.$$

[84]Assumption (4) (i.e. $p_N > 0$) excludes the realization of one of the latter two situations. We analyze them here only with the purpose of revealing the importance of adverse selection costs.

The denominator of the buy price (and at the same time of the assessment P_{BMMt}) is however positive:

$$q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} + (2p_{t-1} - 1) \cdot p_{PI} + 1 \ = \ 2[u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1 - p_{t-1})] \ \geq \ 0,$$

and accordingly, the second partial derivative negative:

$$\frac{\partial^2 X_{Bt}}{\partial q_{HIIt}^2} = \frac{\partial^2 P_{BMMt}}{\partial q_{HIIt}^2} \ = \ \frac{-2p_{t-1}^2 \cdot (1 - p_{t-1}) \cdot p_{II}^2 \cdot [1 - p_{PI} - p_{II} \cdot (1 - q_{LIIt})]}{[q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI} + 1]^3} \leq 0.$$

We can analogously compute the two partial derivatives for the sell price with regard to q_{HIIt}:

$$\begin{aligned}
\frac{\partial X_{St}}{\partial q_{HIIt}} &= \frac{\partial P_{SMMt}}{\partial q_{HIIt}} = \frac{-p_{t-1} \cdot (1 - p_{t-1}) \cdot p_{II} \cdot [(1 - q_{LIIt}) \cdot p_{II} + p_{PI} + 1]}{[-q_{HIIt} \cdot p_{t-1} \cdot p_{II} + (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} + (1 - 2p_{t-1}) \cdot p_{PI} + 1]^2} \\
\frac{\partial^2 X_{St}}{\partial q_{HIIt}^2} &= \frac{\partial^2 P_{SMMt}}{\partial q_{HIIt}^2} = \frac{-2p_{t-1}^2 \cdot (1 - p_{t-1}) \cdot p_{II}^2 \cdot [(1 - q_{LIIt}) \cdot p_{II} + p_{PI} + 1]}{[-q_{HIIt} \cdot p_{t-1} \cdot p_{II} + (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} + (1 - 2p_{t-1}) \cdot p_{PI} + 1]^3}.
\end{aligned}$$

With all probabilities lying in the interval $[0, 1]$ and

$$\begin{aligned}
-q_{HIIt} \cdot p_{t-1} \cdot p_{II} + (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (2p_{t-1} - 1) \cdot p_{PI} + 1 \ &= \\
&= \ 2[(1 - u_{MMt}) \cdot p_{t-1} + (1 - v_{MMt}) \cdot (1 - p_{t-1})] \geq 0,
\end{aligned}$$

we obtain the following negative expressions:

$$\begin{aligned}
\frac{\partial X_{St}}{\partial q_{HIIt}} &\leq 0 \\
\frac{\partial^2 X_{St}}{\partial q_{HIIt}^2} &\leq 0.
\end{aligned}$$

With the exception of the case when $p_{t-1} \in \{0, 1\}$, the prices do not reach any local extremal values.
By means of the previous formulas we can determine the sign of the partial derivatives of the bid-ask spread with regard to the probability q_{HIIt}:

$$\frac{\partial S_t}{\partial q_{HIIt}} = \frac{\partial X_{Bt}}{\partial q_{HIIt}} - \frac{\partial X_{St}}{\partial q_{HIIt}} \ \geq \ 0.$$

In order to obtain the second partial spread derivative: $\dfrac{\partial^2 S_t}{\partial q_{HIIt}^2} \ = \ \dfrac{\partial^2 X_{Bt}}{\partial q_{HIIt}^2} - \dfrac{\partial^2 X_{St}}{\partial q_{HIIt}^2}$,

we introduce the following notation:

$$\begin{aligned}
a &\equiv q_{HIIt} \cdot p_{t-1} \cdot p_{II} \\
b &\equiv (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \\
c &\equiv (1 - 2p_{t-1}) \cdot p_{PI} \\
\alpha &\equiv a - b - c.
\end{aligned} \tag{26}$$

In so doing we have:

$$\frac{\partial^2 S_t}{\partial q_{HIIt}^2} \ = \ \frac{4p_{t-1}^2 \cdot (1 - p_{t-1}) \cdot p_{II}^2}{(1 - \alpha)^3 \cdot (1 + \alpha)^3} \cdot \{\alpha \cdot (3 + \alpha^2) + [p_{PI} + p_{II} \cdot (1 - q_{LIIt})] \cdot (1 + 3\alpha^2)\},$$

with $\alpha \ = \ q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI}$.
The previously proved relations (see the computation of the price-derivatives) $1 - \alpha \geq 0$ and $1 + \alpha \geq 0$ result in:

- for $\alpha \ \geq \ 0$, is $\dfrac{\partial^2 S_t}{\partial q_{HIIt}^2} \ \geq \ 0$;

- for $\alpha < 0$, has $\dfrac{\partial^2 S_t}{\partial q^2_{HIIt}}$ an inflection point in: $-\alpha \cdot (3+\alpha^2) = [p_{PI} + p_{II} \cdot (1 - q_{LIIt})] \cdot (1 + 3\alpha^2)$.
  - as long as $-\alpha \cdot (3+\alpha^2) \leq [p_{PI} + p_{II} \cdot (1 - q_{LIIt})] \cdot (1 + 3\alpha^2)$, we have $\dfrac{\partial^2 S_t}{\partial q^2_{HIIt}} \geq 0$;
  - for $-\alpha \cdot (3+\alpha^2) > [p_{PI} + p_{II} \cdot (1 - q_{LIIt})] \cdot (1 + 3\alpha^2)$, we have $\dfrac{\partial^2 S_t}{\partial q^2_{HIIt}} \leq 0$.

  In other words, the bid-ask spread is concave, if and only if α is negative and small. Because α is directly proportional to q_{HIIt}, this condition is fulfilled only for sufficiently low values of the informational accuracy q_{HIIt}.

  Moreover, $\alpha = 2[u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1 - p_{t-1})] - 1 \; ; 2P(x_t = B|h_{t-1}) - 1$. Accordingly, we can give a second interpretation of the condition for a concave spread evolution: α reaches low values (thus making the bid-ask spread turn to concave) for low buy probabilities (and high sell probabilities, respectively), taking into account past information $P(x_t = B|h_{t-1})$.

The bid-ask spread consequently exhibits a variable variation speed, subject to the model parameters. The condition for a convex spread evolution:

$$\alpha \geq 0 \quad \equiv \quad q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI} \geq 0$$

is more probably fulfilled the higher the probability q_{LIIt} is (other things being equal).

Proposition 2.2. - Proof:
An analogous computation (as for the proof of Proposition 2.1.) results in:

$$\begin{aligned}
\frac{\partial X_{Bt}}{\partial q_{LIIt}} &= \frac{\partial P_{BMMt}}{\partial q_{LIIt}} = \frac{(-p_{t-1}) \cdot (1 - p_{t-1}) \cdot p_{II} \cdot (q_{HIIt} \cdot p_{II} + p_{PI} + 1)}{[q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI} + 1]^2} \\
\frac{\partial^2 X_{Bt}}{\partial q^2_{LIIt}} &= \frac{\partial^2 P_{BMMt}}{\partial q^2_{LIIt}} = \frac{2p_{t-1} \cdot (1 - p_{t-1})^2 \cdot p^2_{II} \cdot (q_{HIIt} \cdot p_{II} + p_{PI} + 1)}{[q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI} + 1]^3}.
\end{aligned}$$

Because all variables take values in the interval $[0, 1]$ and the denominator of the second partial derivative is positive (as previously demonstrated in the Proposition 2.1.), we have:

$$\begin{aligned}
\frac{\partial X_{Bt}}{\partial q_{LIIt}} &\leq 0 \\
\frac{\partial^2 X_{Bt}}{\partial q^2_{LIIt}} &\geq 0.
\end{aligned}$$

If p_{t-1} takes no extreme values (0 or 1), there is no local minimal value of the buy price subject to q_{LIIt}, because $q_{HIIt} \cdot p_{II} + p_{PI} + 1 > 0$.

For the sell price we obtain:

$$\begin{aligned}
\frac{\partial X_{St}}{\partial q_{LIIt}} &= \frac{\partial P_{SMMt}}{\partial q_{LIIt}} = \frac{p_{t-1} \cdot (1 - p_{t-1}) \cdot p_{II} \cdot (1 - q_{HIIt} \cdot p_{II} - p_{PI})}{[-q_{HIIt} \cdot p_{t-1} \cdot p_{II} + (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} + (1 - 2p_{t-1}) \cdot p_{PI} + 1]^2} \\
\frac{\partial^2 X_{St}}{\partial q^2_{LIIt}} &= \frac{\partial^2 P_{SMMt}}{\partial q^2_{LIIt}} = \frac{2p_{t-1} \cdot (1 - p_{t-1})^2 \cdot p^2_{II} \cdot (1 - q_{HIIt} \cdot p_{II} - p_{PI})}{[-q_{HIIt} \cdot p_{t-1} \cdot p_{II} + (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} + (1 - 2p_{t-1}) \cdot p_{PI} + 1]^3}.
\end{aligned}$$

Because all probabilities lie in the interval $[0, 1]$, there results $0 \leq q_{HIIt} \cdot p_{II} \leq p_{II}$ and therewith $(1 - q_{HIIt} \cdot p_{II} - p_{PI}) \geq (1 - p_{II} - p_{PI}) = p_N \geq 0$. The two first partial derivatives are consequently positive:

$$\begin{aligned}\frac{\partial X_{St}}{\partial q_{LIIt}} &\geq 0\\ \frac{\partial^2 X_{St}}{\partial q^2_{LIIt}} &\geq 0.\end{aligned}$$

Because $1 - q_{HIIt} \cdot p_{II} - p_{PI} = p_N + p_{II} \cdot (1 - q_{HIIt})$ and according to the Assumption (4) $p_N > 0$, the sell price exhibits no extreme value for $p_{t-1} \neq \{0, 1\}$.

With regard to the spread evolution subject to the c.p. variation of the probability q_{LIIt}, we have:

$$\frac{\partial S_t}{\partial q_{LIIt}} = \frac{\partial X_{Bt}}{\partial q_{LIIt}} - \frac{\partial X_{St}}{\partial q_{LIIt}} \leq 0.$$

For the computation of the second partial derivative we adopt the notations (26) from the proof of Proposition 2.1. Accordingly, we have:

$$\frac{\partial^2 S_t}{\partial q^2_{LIIt}} = \frac{4p_{t-1} \cdot (1-p_{t-1})^2 \cdot p^2_{II}}{(1-\alpha)^3 \cdot (1+\alpha)^3} \cdot \{-\alpha \cdot (3+\alpha^2) + (p_{PI} + p_{II} \cdot q_{HIIt}) \cdot (1+3\alpha^2)\},$$

with $\alpha = q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI}$.
Because $1 - \alpha \geq 0$ and $1 + \alpha \geq 0$ (i.e. $-1 \leq \alpha \leq 1$), we obtain:

- for $\alpha \leq 0$, is $\frac{\partial^2 S_t}{\partial q^2_{LIIt}} \geq 0$;
- for $\alpha > 0$, has $\frac{\partial^2 S_t}{\partial q^2_{LIIt}}$ an inflection point in: $\alpha \cdot (3 + \alpha^2) = (p_{PI} + p_{II} \cdot q_{HIIt}) \cdot (1 + 3\alpha^2)$.
  - as long as $\alpha \cdot (3 + \alpha^2) \leq (p_{PI} + p_{II} \cdot q_{HIIt}) \cdot (1 + 3\alpha^2)$, is $\frac{\partial^2 S_t}{\partial q^2_{LIIt}} \geq 0$;
  - for $\alpha \cdot (3 + \alpha^2) > (p_{PI} - p_{II} \cdot q_{HIIt}) \cdot (1 + 3\alpha^2)$, in turn $\frac{\partial^2 S_t}{\partial q^2_{LIIt}} \leq 0$.

  Therefore, the bid-ask spread runs concavely only for a positive and sufficiently high α. By virtue of the direct proportionality between α and q_{LIIt} (other things being equal) the bid-ask spread reaches the inflection point only for high values of q_{LIIt} (as an inverse measure of the informational accuracy).

  By means of the relation $\alpha = 2[u_{MMt} \cdot p_{t-1} + v_{MMt} \cdot (1 - p_{t-1})] - 1 = 2P(x_t = B|h_{t-1}) - 1$, we can provide another interpretation of this result: the parameter α takes high positive values (thus rendering the bid-ask spread concave) for high buy probabilities (and for low sell probabilities, respectively), taking into account the past information $P(x_t = B|h_{t-1})$.

The bid-ask spread changes consequently with variable speed, subject to the model parameters. The condition for a convex evolution:

$$\alpha \leq 0 \quad \equiv \quad q_{HIIt} \cdot p_{t-1} \cdot p_{II} - (1 - q_{LIIt}) \cdot (1 - p_{t-1}) \cdot p_{II} - (1 - 2p_{t-1}) \cdot p_{PI} \geq 0$$

is all the more probably fulfilled for given values of the model variables, the lower the probability q_{LIIt} is.

Proposition 2.3. - Proof:

Once again, we compute the two partial derivatives of the prices subject to p_{II}:

$$\begin{aligned}
\frac{\partial X_{Bt}}{\partial p_{II}} &= \frac{\partial P_{BMMt}}{\partial p_{II}} = \frac{p_{t-1}\cdot(1-p_{t-1})\cdot[q_{HIIt}(1-p_{PI})+(1-q_{LIIt})\cdot(1+p_{PI})]}{[q_{HIIt}\cdot p_{t-1}\cdot p_{II}-(1-q_{LIIt})\cdot(1-p_{t-1})\cdot p_{II}+(2p_{t-1}-1)\cdot p_{PI}+1]^2}\\
\frac{\partial^2 X_{Bt}}{\partial p_{II}^2} &= \frac{\partial^2 P_{BMMt}}{\partial p_{II}^2} = \frac{\partial X_{Bt}}{\partial p_{II}}\cdot\frac{-2\cdot[-p_{t-1}\cdot(1+q_{HIIt}-q_{LIIt})-(1-q_{LIIt})]}{[q_{HIIt}\cdot p_{t-1}\cdot p_{II}-(1-q_{LIIt})\cdot(1-p_{t-1})\cdot p_{II}+(2p_{t-1}-1)\cdot p_{PI}+1]}.
\end{aligned}$$

Because all probabilities lie in the interval $[0,1]$ and the denominator of the second partial derivative is positive (see the proof of Proposition 2.1), we have:

$$\begin{aligned}
\frac{\partial X_{Bt}}{\partial p_{II}} &\geq 0\\
\frac{\partial^2 X_{Bt}}{\partial p_{II}^2} &\geq 0.
\end{aligned}$$

Analogously, we ascertain:

$$\begin{aligned}
\frac{\partial X_{St}}{\partial p_{II}} &= \frac{\partial P_{SMMt}}{\partial p_{II}} = \frac{p_{t-1}\cdot(1-p_{t-1})\cdot[-q_{HIIt}\cdot(1+p_{PI})-(1-q_{LIIt})\cdot(1-p_{PI})]}{[-q_{HIIt}\cdot p_{t-1}\cdot p_{II}+(1-q_{LIIt})\cdot(1-p_{t-1})\cdot p_{II}+(1-2p_{t-1})\cdot p_{PI}+1]^2}\\
\frac{\partial^2 X_{St}}{\partial p_{II}^2} &= \frac{\partial^2 P_{SMMt}}{\partial p_{II}^2} = \frac{\partial X_{St}}{\partial p_{II}}\cdot\frac{-2\cdot[p_{t-1}\cdot(1-q_{HIIt}+q_{LIIt})+(1-q_{LIIt})]}{[-q_{HIIt}\cdot p_{t-1}\cdot p_{II}+(1-q_{LIIt})\cdot(1-p_{t-1})\cdot p_{II}+(1-2p_{t-1})\cdot p_{PI}+1]}.
\end{aligned}$$

Because all probabilities take values in the interval $[0,1]$ and the denominator of the second partial derivative is positive (see Proposition 2.1.), it results in:

$$\begin{aligned}
\frac{\partial X_{St}}{\partial p_{II}} &\leq 0\\
\frac{\partial^2 X_{St}}{\partial p_{II}^2} &\leq 0.
\end{aligned}$$

Moreover, the prices take no extreme values, as long as $p_{t-1} \neq \{0,1\}$.

The spread variation can be ascertained by means of the partial derivatives of the ask and bid prices:

$$\begin{aligned}
\frac{\partial S_t}{\partial p_{II}} &= \frac{\partial X_{Bt}}{\partial p_{II}} - \frac{\partial X_{St}}{\partial p_{II}} \geq 0\\
\frac{\partial^2 S_t}{\partial p_{II}^2} &= \frac{\partial^2 X_{Bt}}{\partial p_{II}^2} - \frac{\partial^2 X_{St}}{\partial p_{II}^2} \geq 0.
\end{aligned}$$

Proposition 3. - Proof:

Both for $[q_{HIIt}=0$ and $q_{LIIt}=1]$, and $[p_{II}=0]$ the buy and sell prices are the same:

$$\begin{aligned}
X_{Bt} &= \frac{p_{t-1}\cdot(p_{PI}+1)}{(2p_{t-1}-1)\cdot p_{PI}+1}\\
X_{St} &= \frac{p_{t-1}\cdot(-p_{PI}+1)}{(1-2p_{t-1})\cdot p_{PI}+1}.
\end{aligned}$$

Proposition 4. - Proof:

For $p_{t-1} = 0$ we have: $X_{Bt} = X_{St} = V_L = 0$, while for $p_{t-1} = 1$ it results in: $X_{Bt} = X_{St} = V_H = 1$.

Proposition 5.1. - Proof:
Because $\dfrac{\partial \rho_{IIt}^{V_H}}{\partial q_{HIIt}} = \dfrac{-q_{LIIt} \cdot p_{t-1} \cdot (1-p_{t-1})}{[q_{HIIt} \cdot p_{t-1} + q_{LIIt} \cdot (1-p_{t-1})]^2}$, we have: $\dfrac{\partial \rho_{IIt}^{V_H}}{\partial q_{HIIt}} \leq 0$.
Furthermore: $\dfrac{\partial^2 \rho_{IIt}^{V_H}}{\partial q_{HIIt}^2} = \dfrac{2q_{LIIt} \cdot p_{t-1}^2 \cdot (1-p_{t-1})}{[q_{HIIt} \cdot p_{t-1} + q_{LIIt} \cdot (1-p_{t-1})]^3} \geq 0$.

Analogously we have:

$$\begin{aligned}
\frac{\partial \rho_{IIt}^{V_L}}{\partial q_{HIIt}} &= \frac{-(1-q_{LIIt}) \cdot p_{t-1} \cdot (1-p_{t-1})}{[(1-q_{HIIt}) \cdot p_{t-1} + (1-q_{LIIt}) \cdot (1-p_{t-1})]^2} \leq 0 \\
\frac{\partial^2 \rho_{IIt}^{V_L}}{\partial q_{HIIt}^2} &= \frac{-2(1-q_{LIIt}) \cdot p_{t-1}^2 \cdot (1-p_{t-1})}{[(1-q_{HIIt}) \cdot p_{t-1} + (1-q_{LIIt}) \cdot (1-p_{t-1})]^3} \leq 0.
\end{aligned}$$

Excepting the extreme situations with $p_{t-1} \in \{0,1\}$, the two misperceptions do not reach any extreme values. The variation of the second misperception form (with regard to the appropriate action) can be easily ascertained from the definition formulas.

Proposition 5.2. - Proof:

$$\begin{aligned}
\frac{\partial \rho_{IIt}^{V_H}}{\partial q_{LIIt}} &= \frac{q_{HIIt} \cdot p_{t-1} \cdot (1-p_{t-1})}{[q_{HIIt} \cdot p_{t-1} + q_{LIIt} \cdot (1-p_{t-1})]^2} \geq 0 \\
\frac{\partial^2 \rho_{IIt}^{V_H}}{\partial q_{LIIt}^2} &= \frac{-2q_{HIIt} \cdot p_{t-1} \cdot (1-p_{t-1})^2}{[q_{HIIt} \cdot p_{t-1} + q_{LIIt} \cdot (1-p_{t-1})]^3} \leq 0
\end{aligned}$$

Analogously, we have:

$$\begin{aligned}
\frac{\partial \rho_{IIt}^{V_L}}{\partial q_{LIIt}} &= \frac{(1-q_{HIIt}) \cdot p_{t-1} \cdot (1-p_{t-1})}{[(1-q_{HIIt}) \cdot p_{t-1} + (1-q_{LIIt}) \cdot (1-p_{t-1})]^2} \geq 0 \\
\frac{\partial^2 \rho_{IIt}^{V_L}}{\partial q_{LIIt}^2} &= \frac{2(1-q_{HIIt}) \cdot p_{t-1} \cdot (1-p_{t-1})^2}{[(1-q_{HIIt}) \cdot p_{t-1} + (1-q_{LIIt}) \cdot (1-p_{t-1})]^3} \geq 0.
\end{aligned}$$

According to the definition the sell misperception ρ_{IIt}^{S} depends positively and linearly on the c.p. variation of the probability q_{LIIt}.

Section 5 - Proof:
According to the law of iterated expectations we obtain at t the following estimation of the prices at $t+1$:

$$\mathbf{E}[\mathbf{X_{t+1}}|\mathbf{h_t}] = E[E[V_{t+1}|h_{t+1}]|h_t] = E[V_{t+1}|h_t] = V_H \cdot P(V_{t+1} = V_H|h_t) + V_L \cdot P(V_{t+1} = V_L|h_t) = \mathbf{p_t}.$$

The contemporaneous price takes the following value:

$$X_t = E[V_t|h_t] = P(V_t = V_H|h_t).$$

Due to Assumption (9) (see footnote 41) (i.e. the independence of the buys and sells) and to the fact that the market maker fixes the prices accounting only for the past information and the contemporaneous actions of the investors, there results:

$$\begin{aligned}
P(V_t = V_H|h_t) &= P(V_t = V_H|x_t = B \vee S, h_{t-1}) = \frac{P(V_t = V_H, x_t = B \vee S|h_{t-1})}{P(x_t = B \vee S|h_{t-1})} \\
&= \frac{P(V_t = V_H, x_t = B|h_{t-1}) + P(V_t = V_H, x_t = S|h_{t-1})}{P(x_t = B|h_{t-1}) + P(x_t = S|h_{t-1})} \\
&= \frac{P(V_t = V_H|x_t = B, h_{t-1}) \cdot P(x_t = B|h_{t-1}) + P(V_t = V_H|x_t = S, h_{t-1}) \cdot P(x_t = S|h_{t-1})}{1} \\
&= u_{MMt} \cdot p_{t-1} + (1 - u_{MMt}) \cdot p_{t-1} = p_{t-1},
\end{aligned}$$

that means: $\mathbf{X_t = p_{t-1}}$.
The *prices* are consequently *martingales* (i.e. $E[X_{t+1}|h_t] = X_t$), only if $p_t = p_{t-1} = \ldots = p_0$, i.e. *for a constant* p_{t-1}.

According to Assumption (10) ($p_{t-1} = P(V_t = V_H|h_{t-1})$) and to the previous formulas ($X_t = p_{t-1}$), the prices are, however, based only upon past information. The market can therefore be only weak-form efficient (and not semi-strong efficient, as in other approaches).

B Appendix - Graphics

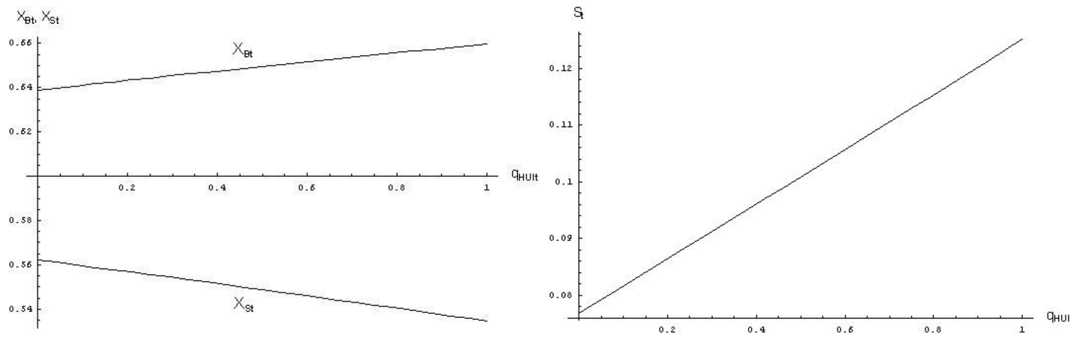

Figure 1: The ask and bid price, and the spread, subject to the accuracy of the method-based information in a good economic situation, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{LIIt} = 0.4$ and $p_{II} = 0.1$

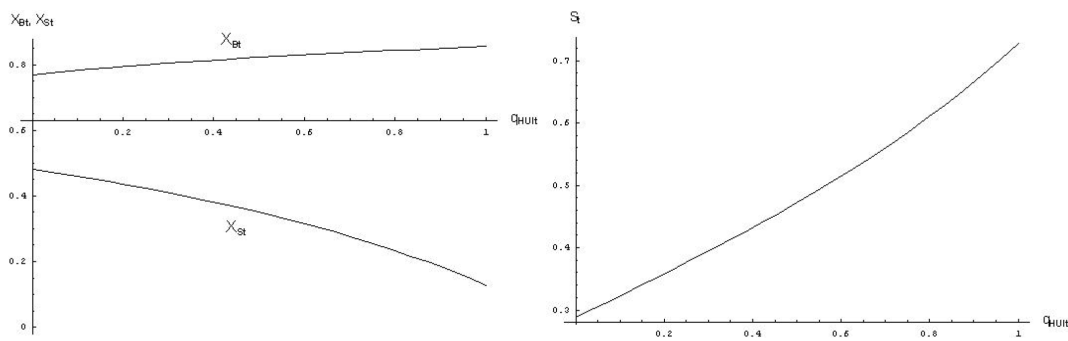

Figure 2: The ask and bid price, and the spread, subject to the accuracy of the method-based information in a good economic situation, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{LIIt} = 0.4$ and $p_{II} = 0.8$

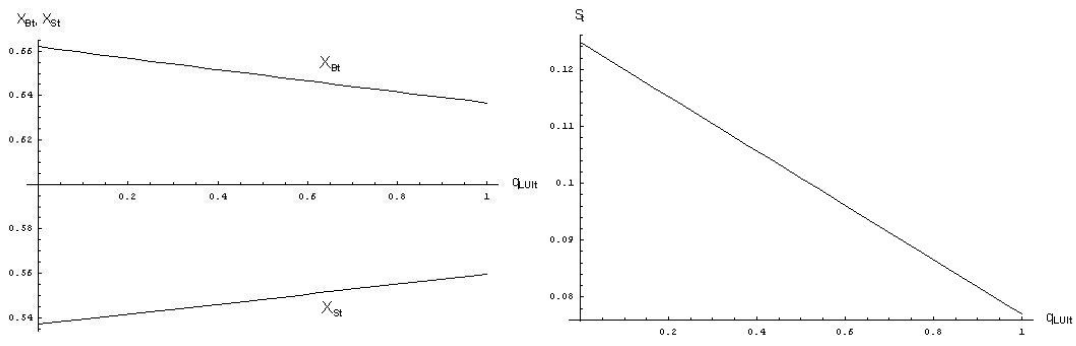

Figure 3: The ask and bid price, and the spread, subject to the accuracy of the method-based information in a bad economic situation, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{HIIt} = 0.6$ and $p_{II} = 0.1$

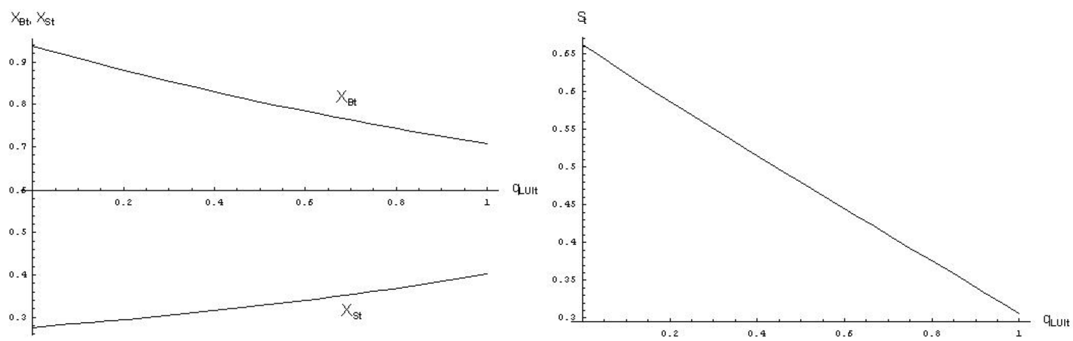

Figure 4: The ask and bid price, and the spread, subject to the accuracy of the method-based information in a bad economic situation, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{HIIt} = 0.6$ and $p_{II} = 0.8$

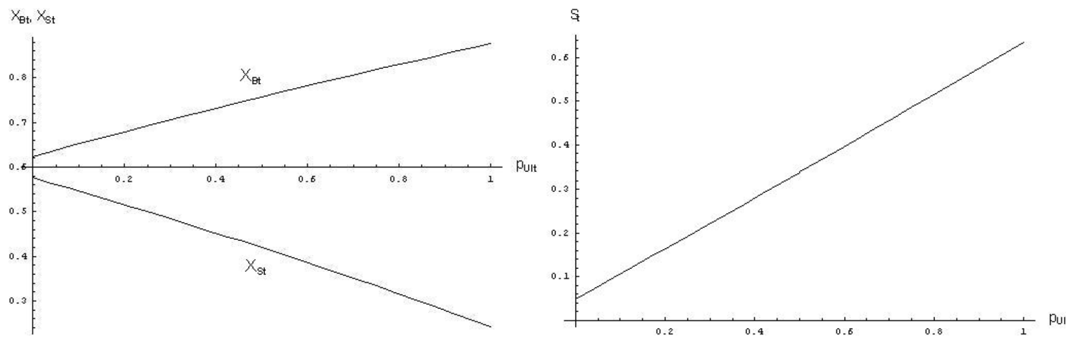

Figure 5: The ask and bid price, and the spread, subject to the fraction of the users of practical decision rules, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{HIIt} = 0.6$ and $q_{LIIt} = 0.4$

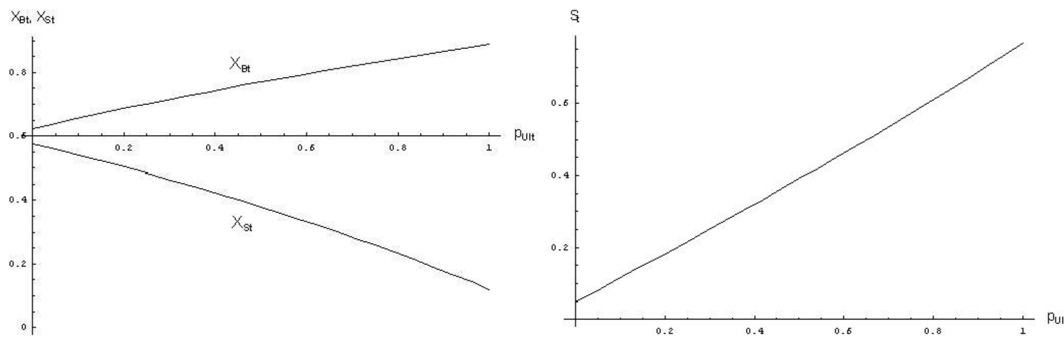

Figure 6: The ask and bid price, and the spread, subject to the fraction of the users of practical decision rules, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$, $q_{HIIt} = 0.8$ and $q_{LIIt} = 0.4$

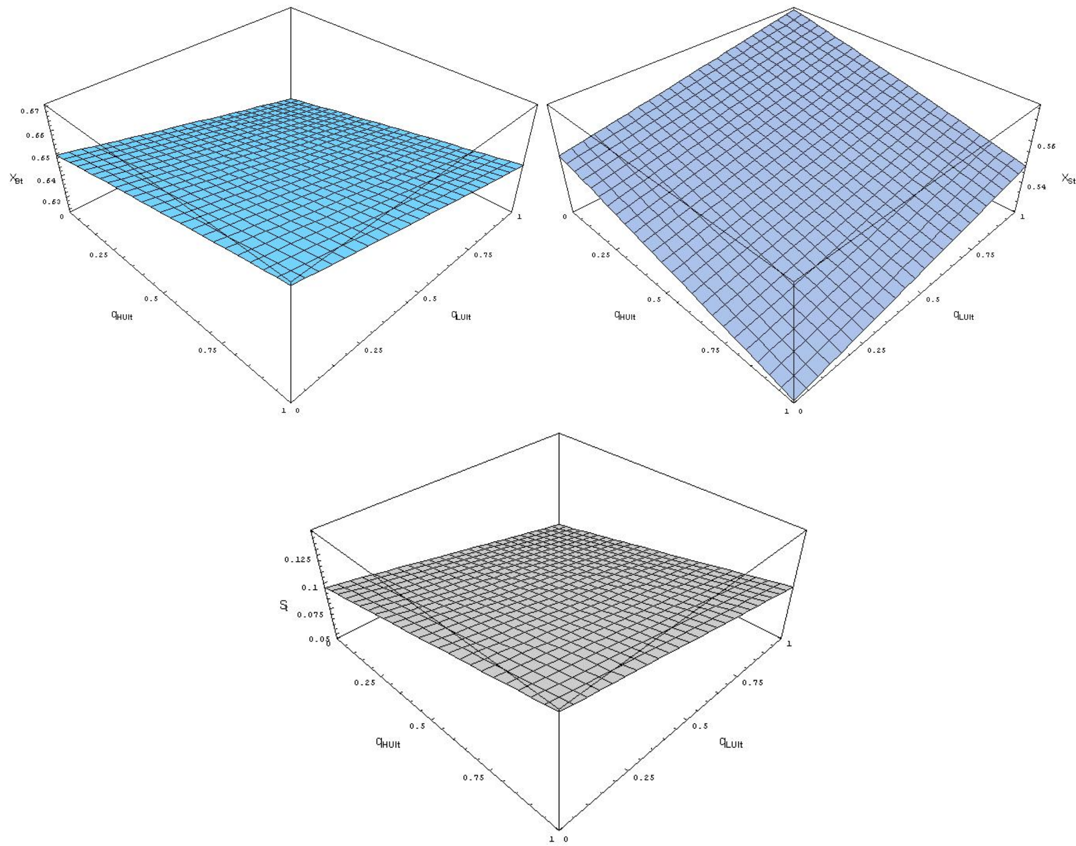

Figure 7: The ask and bid price, and the spread, subject to the accuracy of the method-based information, in the particular case with : $p_{t-1} = 0.6$, $p_{PI} = 0.05$ and $p_{II} = 0.1$

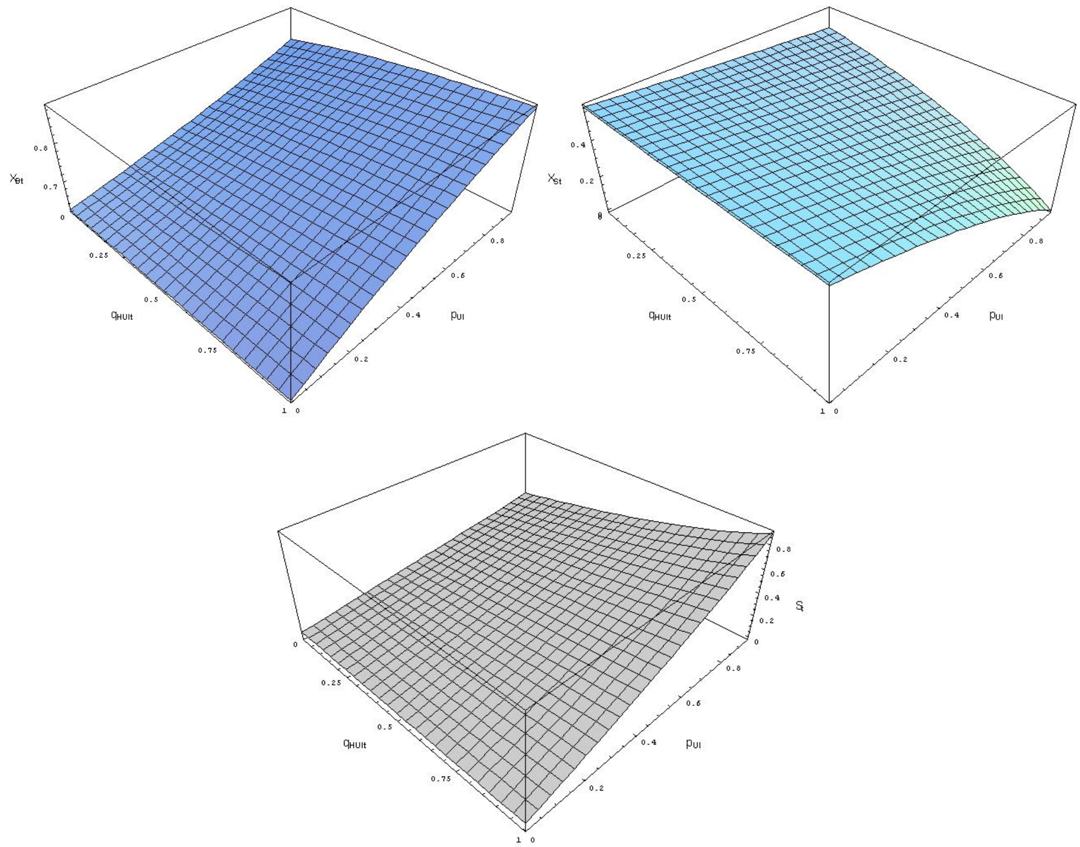

Figure 8: The ask and bid price, and the spread, subject to the accuracy of the method-based information in a good economic situation and to the fraction of the users or practical decision rules, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$ and $q_{LIIt} = 0.4$

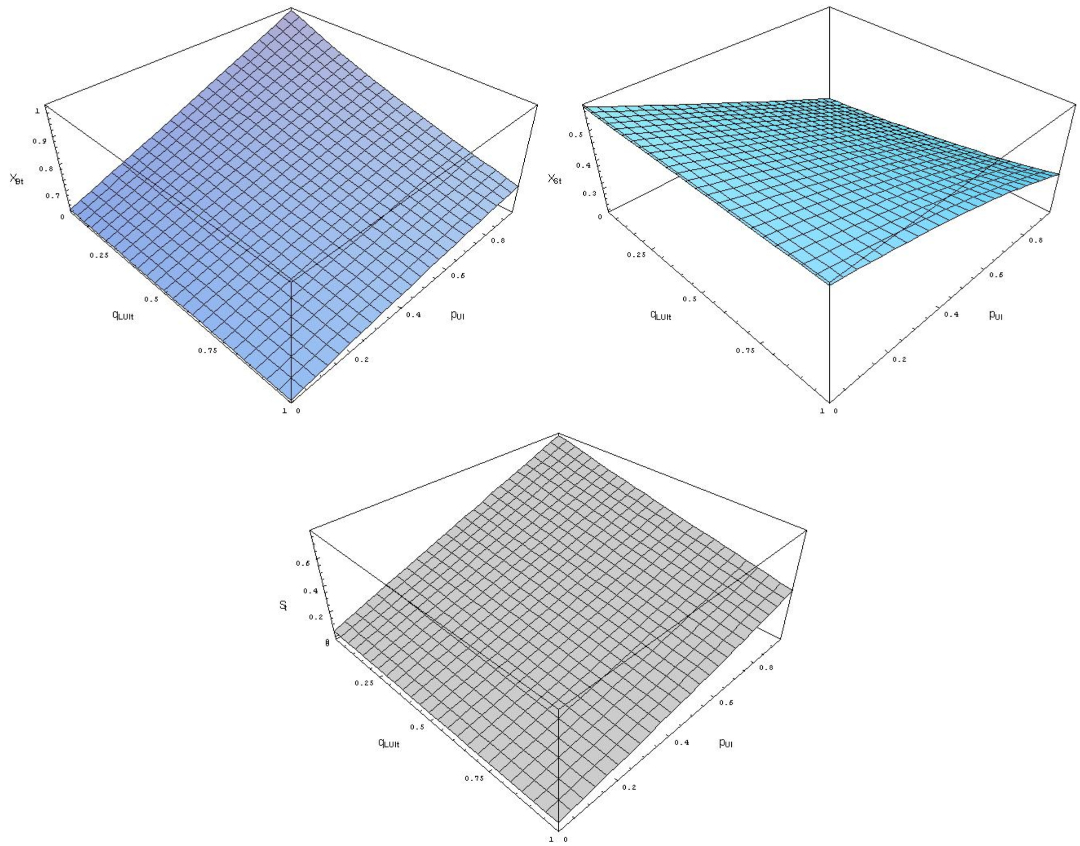

Figure 9: The ask and bid price, and the spread, subject to the accuracy of the method-based information in a bad economic situation and to the fraction of the users or practical decision rules, in the particular case with: $p_{t-1} = 0.6$, $p_{PI} = 0.05$ and $q_{HIIt} = 0.6$

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